Filter-And-Forward Relay Design for OFDM Systems for Quality-of-Service Enhancement

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***Abstract*—In this paper, the filter-and-forward (FF) relay design for OFDM communication systems is considered to enhance the system performance over the conventional amplify-and-forward (AF) relaying scheme. The considered design criterion in this paper is to maximize the worst subcarrier channel signal-to-noise ratio (SNR) subject to the total relay transmit power constraint in order to improve the overall transmission quality-of-service. It is shown, by exploiting the eigen-property of circulant matrices and the structure of Toeplitz matrices, that the considered problem reduces to a semi-definite programming (SDP) problem. Numerical results are provided to validate our design method and the numerical results show that the proposed FF relay outperforms the AF scheme significantly.**

I. Introduction

Relays have become a major research topic in wireless communications recently since they can increase the network coverage and enhance the system QoS in current and future wireless communication systems. Indeed, the LTE-Advanced standard adopts relays for coverage extension and QoS improvement [2]. Among several relaying schemes under consideration, the AF scheme is easy to implement and appropriate for cheap relay operation in the transparent mode in which destination nodes do not know the existence of the relay [2]. Recently, there have been some research works to extend the AF scheme to a generalized filter-and-forward (FF) scheme, in order to achieve better performance than the AF scheme with tolerable complexity increase [3]-[6], and it is shown that the FF scheme can outperform the AF scheme significantly. However, most of the previous results on the FF scheme focused on single-carrier transmission, while most of the current wireless standards are based on multi-carrier OFDM transmission.

In this paper, we propose a novel relay scheme, i.e. FF relaying for OFDM transmission. Unlike most OFDM relays which OFDM-demodulate the received signal, amplify or decode the demodulated signal, OFDM-remodulate the processed signal and forward the remodulated OFDM signal to the destination [7], [8], in the proposed scheme the FF relay FIR-filters the incoming signal at the chip rate of the OFDM modulation and directly forwards the filtered signal to the destination. Thus, the necessity of OFDM processing at the relay is removed while the system performance can be enhanced over the AF scheme by the proposed FF scheme. Among several FF relay design criteria, we consider QoS improvement under the scenario of single-input single-output (SISO) communication. That is, we design the FF relay filter in order to maximize the worst subcarrier channel SNR subject to a total relay transmit power constraint. By exploiting the eigen-property of circulant matrices and the structure of Toeplitz filtering matrices, we show that the problem can be reformulated as an SDP problem based on a semi-definite relaxation (SDR) approach. Numerical results show that the proposed FF relay significantly outperforms conventional AF relays under the criterion.

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II. System Model

In this paper, we consider a single-input single-output (SISO) full-duplex relay network consisting of a source node, a relay node and a destination node. We assume that there is no direct link between the source and destination nodes and that both the source-to-relay (SR) link and the relay-to-destination (RD) link are inter-symbol interference (ISI) channels modelled as FIR filters. Considering the transparent operation of the proposed FF scheme, we assume that the SR channel state information (CSI) is known to the relay whereas the RD CSI is unknown to the relay but the RD channel distribution information (CDI) is known to the relay. The FF relay performs FIR filtering on the incoming signal at the chip rate of the OFDM modulation and transmits the filtered signal directly to the destination.

Now, let us describe our system model in detail. At the source, the OFDM symbol vector of size N is given by $\mathbf{s} := [s[0], s[1], \cdots, s[N-1]]^T$, where each data symbol is assumed to be a zero-mean independent complex Gaussian random variable, i.e., $s[k] \sim \mathcal{CN}(0, P_{s,k})$ for $k = 0, 1, \cdots, N-1$. After normalized inverse discrete Fourier transform (IDFT), the time-domain signal vector $\mathbf{x}_s$ at the source is obtained as

$$\begin{aligned}\underbrace{\begin{bmatrix} x_s[0] \\ x_s[1] \\ \vdots \\ x_s[N\text{-}1] \end{bmatrix}}_{=:\mathbf{x}_s} &= \underbrace{\frac{1}{\sqrt{N}}\begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega_N & \cdots & \omega_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_N^{N-1} & \cdots & \omega_N^{(N-1)^2} \end{bmatrix}}_{=:\mathbf{W}_N} \underbrace{\begin{bmatrix} s[0] \\ s[1] \\ \vdots \\ s[N\text{-}1] \end{bmatrix}}_{=\mathbf{s}}, \\ &= [\mathbf{w}_0^T\mathbf{s},\ \mathbf{w}_1^T\mathbf{s},\ \cdots,\ \mathbf{w}_{N-1}^T\mathbf{s}]^T, \end{aligned} \quad (1)$$

where $\omega_N = e^{j\frac{2\pi}{N}}$ and $\mathbf{w}_{k-1}^T$ is the k-th row of the normalized IDFT matrix $\mathbf{W}_N$ for $k = 1, \cdots, N$. The vector $\mathbf{x}_s$ is attached by a cyclic prefix with length L_{CP}, i.e.,

$$\tilde{x}_s[n] = \begin{cases} x_s[n], & n = 0, 1, \cdots, N-1, \\ x_s[N+n], & n = -1, -2, \cdots, -L_{CP}, \end{cases} \quad (2)$$

and the cyclic prefix attached signal $\tilde{x}_s[n]$ is transmitted from the source to the relay through the SR channel. Then, the baseband signal received at the relay is expressed as

$$y_r[n] = \sum_{l=0}^{L_f-1} f_l \tilde{x}_s[n-l] + n_r[n], \quad (3)$$

where $\mathbf{f} = [f_0, f_1, \cdots, f_{L_f-1}]^T$ is the channel coefficient vector of the SR FIR channel known to the relay; L_f is its channel length; and $n_r[n]$ is the zero-mean additive white Gaussian noise with variance σ_r^2 at the relay, i.e., $n_r[n] \sim \mathcal{CN}(0, \sigma_r^2)$. At the FF relay, the received signal $y_r[n]$ is FIR filtered and then transmitted to the destination. Thus, the output signal at the relay at chip time n is expressed as

$$y_t[n] = \sum_{l=0}^{L_r-1} r_l \, y_r[n-l], \quad (4)$$

where $\mathbf{r} = [r_0, r_1, \cdots, r_{L_r-1}]^T$ is the FIR filter coefficient vector at the FF relay and L_r is the order of the FIR filter. Note that the FF relay reduces to an AF relay in the case of $L_r = 1$. The transmitted signal from the relay goes through the RD FIR channel to the destination. Finally, the received signal at the destination is expressed as

$$y_d[n] = \sum_{l=0}^{L_g-1} g_l \, y_t[n-l] + n_d[n], \quad (5)$$

where $\mathbf{g} = [g_0, g_1, \cdots, g_{L_g-1}]^T$ is the RD FIR channel coefficient vector; L_g is the RD FIR channel length; and $n_d[n]$ is zero-mean white Gaussian noise with variance σ_d^2 at the destination. Here, it is assumed that each RD channel tap coefficient is independent and identically distributed (i.i.d.) according to $g_l \sim \mathcal{CN}(0, \sigma_g^2)$ and the realization $\{g_l, \ l = 0, 1, \cdots, L_g - 1\}$ is unknown to the relay but its distribution is known to the relay. Stacking the transmitted symbols at the relay and the cyclic prefix portion removed received symbols at the destination, respectively, we have the following vectors:

$$\mathbf{y}_t = \mathbf{RF}\tilde{\mathbf{x}}_s + \mathbf{R}\mathbf{n}_r \text{ and } \mathbf{y}_d = \mathbf{GRF}\tilde{\mathbf{x}}_s + \mathbf{GR}\mathbf{n}_r + \mathbf{n}_d, \quad (6)$$

where

$$\mathbf{y}_d = [y_d[N\text{-}1], \cdots, y_d[0]]^T, \quad (7)$$
$$\mathbf{y}_t = [y_t[N\text{-}1], \cdots, y_t[0], y_t[-1], \cdots, y_t[-L_g+1]]^T, \quad (8)$$
$$\tilde{\mathbf{x}}_s = [x_s[N\text{-}1], \cdots, x_s[0], \cdots, x_s[-L_g - L_r - L_f + 3]]^T, \quad (9)$$
$$\mathbf{n}_r = [n_r[N\text{-}1], \cdots, n_r[0], \cdots, n_r[-L_g - L_r + 2]]^T, \quad (10)$$
$$\mathbf{n}_d = [n_d[N\text{-}1], \cdots, n_d[0]]^T, \quad (11)$$
$$\mathbf{G} = \texttt{Toeplitz}(\mathbf{g}^T, N), \quad (12)$$
$$\mathbf{R} = \texttt{Toeplitz}(\mathbf{r}^T, N + L_g - 1), \quad (13)$$
$$\mathbf{F} = \texttt{Toeplitz}(\mathbf{f}^T, N + L_g + L_r - 2). \quad (14)$$

Here, the notation $\texttt{Toeplitz}(\mathbf{g}^T, N)$ represents a $N \times (N + L_g - 1)$ Toeplitz matrix with N rows and $[\mathbf{g}^T, 0, \cdots, 0]$ as its first row vector, where $\mathbf{g}^T$ is a row vector of size L_g. It is assumed that $L_{CP} \geq L_g + L_r + L_f - 3$ for proper OFDM demodulation. Then, the DFT of the cyclic prefix portion removed received vector of size N at the destination is obtained as

$$\begin{aligned} \hat{\mathbf{y}}_d &= \mathbf{W}_N^H \mathbf{GRF}\tilde{\mathbf{x}}_s + \mathbf{W}_N^H \mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d, \quad (15) \\ &= \mathbf{W}_N^H \mathbf{H}_c \mathbf{W}_N \mathbf{s} + \mathbf{W}_N^H \mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d, \quad (16) \\ &= \mathbf{Ds} + \mathbf{W}_N^H \mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d, \quad (17) \end{aligned}$$

where $\mathbf{W}_N^H$ is the normalized DFT matrix of size N; $\mathbf{H}_c$ is a $N \times N$ circulant matrix induced* from the overall Toeplitz filtering matrix $\mathbf{GRF}$ from the source to the destination; and $\mathbf{D} = \mathbf{W}_N^H \mathbf{H}_c \mathbf{W}_N = \text{diag}(d_0, \cdots, d_{N-1})$ is the eigen-decomposition of $\mathbf{H}_c$. Here, $\text{diag}(d_1, \cdots, d_n)$ means a diagonal matrix with diagonal elements $d_1, \cdots, d_n$.

Although the received signal form (17) is canonical in OFDM transmission, it cannot be directly used for relay filter design in the following section. To facilitate optimization formulation, we here derive useful expressions for the signal and noise parts of (17), based on the following lemma [9]:

Lemma 1: [9] Let $\mathbf{C}$ be an $N \times N$ circulant matrix with the first row $[c(0), c(1), \cdots, c(N-1)]$. Then, the eigenvalues of $\mathbf{C}$ are given by

$$\lambda_k = \sum_{n=0}^{N-1} c(n)\omega_N^{-kn}, \ k = 0, 1, \cdots, N-1,$$

with the corresponding right eigenvectors

$$\boldsymbol{\xi}_k = \frac{1}{\sqrt{N}}[1, \omega_N^{-k}, \omega_N^{-2k}, \cdots, \omega_N^{-(N-1)k}]^T, \ k = 0, 1, \cdots, N-1.$$

Exploiting Lemma 1, we can derive the diagonal elements of $\mathbf{D}$ in (17) only with the knowledge of the first row of $\mathbf{H}_c$ in (16). The first row of $\mathbf{H}_c$ can be obtained from the first row $\tilde{\mathbf{g}}^T\mathbf{RF}$ of $\mathbf{GRF}$, where the $1 \times (N + L_g - 1)$ row vector $\tilde{\mathbf{g}}^T = [\mathbf{g}^T, 0, \cdots, 0]$ is the first row of $\mathbf{G}$, and is simply the first N elements of $\tilde{\mathbf{g}}^T\mathbf{RF}$, i.e., $\tilde{\mathbf{g}}^T\mathbf{RFT}$, where

$$\mathbf{T} = \begin{bmatrix} \mathbf{I}_N \\ \mathbf{0}_{(L_g+L_r+L_f-3)\times N} \end{bmatrix}. \quad (19)$$

and $\mathbf{I}_N$ is the identity matrix of size N. Note that $\mathbf{T}$ is a truncation matrix eliminating the elements of $\tilde{\mathbf{g}}^T\mathbf{RF}$ except the first N elements. Thus, the diagonal elements of $\mathbf{D}$ can be obtained by DFT as

$$[d_0, \cdots, d_{N-1}]^T = \sqrt{N}\mathbf{W}_N^H(\tilde{\mathbf{g}}^T\mathbf{RFT})^T, \quad (20)$$

where $\sqrt{N}\mathbf{W}_N^H$ is the N-point DFT matrix. Thus, the received signal at the destination in the k-th subcarrier is given by

$$\hat{y}_d[k] = \sqrt{N}\mathbf{w}_k^H \mathbf{T}^T \mathbf{F}^T \mathbf{R}^T \tilde{\mathbf{g}} s[k] + \mathbf{w}_k^H \mathbf{GR}\mathbf{n}_r + \mathbf{w}_k^H \mathbf{n}_d, \quad (21)$$

where $\mathbf{w}_k^H$ is the $(k+1)$-th row vector of $\mathbf{W}_N^H$. Consequently, the signal and noise parts of $\hat{y}_d[k]$ are respectively given by

$$\hat{y}_{d,S}[k] = \sqrt{N}\mathbf{w}_k^H \mathbf{T}^T \mathbf{F}^T \mathbf{R}^T \tilde{\mathbf{g}} s[k] \text{ and}$$

*$\mathbf{H}_c$ is obtained by truncating out the elements of $\mathbf{GRF}$ outside the first $N \times N$ positions and moving the lower $(L_g + L_r + L_f - 3) \times (L_g + L_r + L_f - 3)$ elements of the truncated part to the lower left of the untruncated $N \times N$ matrix.

$$\mathbf{E}_1 = [\underbrace{\mathbf{I}_{L_r}, \mathbf{0}_{L_r\times(N+L_g-2)}}_{N+L_g+L_r-2 \text{ columns}}, \underbrace{\mathbf{0}_{L_r\times 1}, \mathbf{I}_{L_r}, \mathbf{0}_{L_r\times(N+L_g-3)}}_{N+L_g+L_r-2 \text{ columns}}, \cdots, \underbrace{\mathbf{0}_{L_r\times(N+L_g-2)}, \mathbf{I}_{L_r}}_{N+L_g+L_r-2 \text{ columns}}]. \tag{18}$$

$$\hat{y}_{d,N}[k] = \mathbf{w}_k^H \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{w}_k^H \mathbf{n}_d \tag{22}$$

for $k = 0, 1, \cdots, N-1$.

III. Filter-and-forward Relay Design: Worst Subcarrier SNR Maximization

In this section, based on the system model described in the previous section, we propose a FF relay design method for QoS improvement. In particular, we consider the maximization of the worst subcarrier SNR subject to a relay power constraint. Such a design criterion is meaningful for OFDM systems since certain subcarrier channels are in deep notch and the QoS of these subcarrier channels are not good. By maximizing the worst subcarrier channel SNR, we can improve the overall transmission QoS. The considered problem is formulated as follows:

$$\max_{\mathbf{r}} \min_{k\in\{0,\cdots,N-1\}} \text{SNR}_k \text{ s.t. } P_r \le P_{r,\max}, \tag{23}$$

where SNR_k is the SNR of the k-th subcarrier and P_r is the relay transmit power. To solve this problem, we need to express the k-th subcarrier SNR and the relay transmit power as functions of the design parameter, i.e., the relay FIR coefficient vector $\mathbf{r}$. First, let us derive the k-th subcarrier SNR. This can be done by exploiting the Toeplitz structure of $\mathbf{R}$ as follows. Based on the signal part in (22), the received signal power in the k-th subcarrier at the destination is obtained as

$$\begin{aligned}
\mathbb{E}\{|\hat{y}_{d,S}[k]|^2\} &= N\mathbb{E}\{\mathbf{w}_k^H \mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}|s[k]|^2\tilde{\mathbf{g}}^H\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\},\\
&= NP_{s,k}\sigma_g^2 \text{tr}(\underbrace{\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T}_{=:\mathbf{K}_k}\mathbf{R}_{L_g}^T\mathbf{R}_{L_g}^*),\\
&= NP_{s,k}\sigma_g^2 \left[\text{vec}(\mathbf{R}^T)\right]^H \tilde{\mathbf{K}}_k \text{ vec}(\mathbf{R}^T)\\
&= NP_{s,k}\sigma_g^2 \mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r},
\end{aligned} \tag{24}$$

where $\mathbf{E}_1$ is given by (18), $\mathbf{R}_{L_g} = \tilde{\mathbf{I}}_{L_g}\mathbf{R}$ is the matrix composed of the first L_g rows of $\mathbf{R}$, $\tilde{\mathbf{K}}_k = \tilde{\mathbf{I}}_{L_g} \otimes \mathbf{K}_k$, and

$$\tilde{\mathbf{I}}_{L_g} := \begin{bmatrix} \mathbf{I}_{L_g} & \mathbf{0}_{L_g\times(N-1)} \\ \mathbf{0}_{(N-1)\times L_g} & \mathbf{0}_{(N-1)\times(N-1)} \end{bmatrix}.$$

Next, consider the received noise power at the k-th subcarrier. Based on the noise part in (22), the noise power can be expressed as

$$\begin{aligned}
\mathbb{E}\{|\hat{y}_{d,N}[k]|^2\} &= \sigma_r^2\text{tr}(\mathbf{R}^H\underbrace{\mathbf{G}^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{G}}_{=:\mathbf{M}_k}\mathbf{R}) + \sigma_d^2,\\
&= \sigma_r^2\left[\text{vec}(\mathbf{R})\right]^H\tilde{\mathbf{M}}_k\text{vec}(\mathbf{R}) + \sigma_d^2,\\
&= \sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r} + \sigma_d^2,
\end{aligned} \tag{25}$$

where $\tilde{\mathbf{M}}_k = \mathbf{I}_{N+L_g+L_r-2}\otimes\mathbf{M}_k$ and $\mathbf{E}_2$ is given by

$$\mathbf{E}_2 = \begin{bmatrix}
\mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T & \mathbf{0}^T & & \cdots & \mathbf{0}^T\\
\mathbf{0}^T & \mathbf{e}_1^T & \mathbf{e}_2^T & & \cdots & \mathbf{e}_{N+L_g-1}^T & \ddots & \mathbf{0}^T\\
\mathbf{0}^T & \ddots & \ddots & & \ddots & \ddots & \ddots & \vdots\\
\vdots & & \ddots & & \ddots & \ddots & \ddots & \mathbf{0}^T\\
\mathbf{0}^T & \cdots & \mathbf{0}^T & & \mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T
\end{bmatrix}.$$

Here, $\mathbf{e}_i^T$ be the i-th row of $\mathbf{I}_{N+L_g-1}$ and the size of each $\mathbf{0}^T$ is $1\times(N+L_g-1)$. Based on (24) and (25), the k-th subcarrier SNR is given in terms of $\mathbf{r}$ as

$$\text{SNR}_k = \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}. \tag{26}$$

Now, consider the relay transmit power P_r. Based on (6), the relay transmit power is obtained in a similar way as

$$\begin{aligned}
\mathbb{E}\{\text{tr}(\mathbf{y}_t\mathbf{y}_t^H)\} &= \text{tr}(\mathbf{R}\mathbf{F}\mathbb{E}\{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s^H\}\mathbf{F}^H\mathbf{R}^H) + \text{tr}(\sigma_r^2\mathbf{R}\mathbf{R}^H),\\
&= \text{tr}(\mathbf{R}\underbrace{(\mathbf{F}\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}_s}\mathbf{F}^H + \sigma_r^2\mathbf{I})}_{=:\boldsymbol{\Pi}}\mathbf{R}^H),\\
&= \left[\text{vec}(\mathbf{R}^H)\right]^H\tilde{\boldsymbol{\Pi}}\text{vec}(\mathbf{R}^H),\\
&= \mathbf{r}^T\mathbf{E}_1\tilde{\boldsymbol{\Pi}}\mathbf{E}_1^H\mathbf{r}^* = \mathbf{r}^H\mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H\mathbf{r},
\end{aligned} \tag{27}$$

where $\tilde{\boldsymbol{\Pi}} = \mathbf{I}_{N+L_g-1}\otimes\boldsymbol{\Pi}$ and $\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}_s} = \mathbb{E}\{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s^H\}$.

Finally, based on (26) and (27), the worst subcarrier SNR maximization problem under the total relay transmit power can be formulated as follows:

$$\max_{\mathbf{r}} \min_{k\in\{0,\cdots,N-1\}} \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2} \tag{28}$$

$$\text{s.t.} \quad \mathbf{r}^H\mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \le P_{r,\max}. \tag{29}$$

With the introduction of a slack variable τ, the above problem can be restated as

$$\begin{aligned}
\max_{\mathbf{r}} \quad & \tau\\
\text{s.t.} \quad & \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2} \ge \tau, \ k = 0,1,\ldots,N-1,\\
& \mathbf{r}^H\mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \le P_{r,\max}.
\end{aligned} \tag{30}$$

In general, this is a non-convex problem. However, the problem can still be solved by using convex optimization techniques. Let $\mathcal{R} := \mathbf{r}\mathbf{r}^H$. Then, by using $\text{tr}(\mathbf{ABC}) = \text{tr}(\mathbf{BCA})$ and relaxing the rank one constraint for $\mathcal{R}$, the problem can be rewritten as

$$\begin{aligned}
\max_{\mathcal{R}} \quad & \tau\\
\text{s.t.} \quad & \text{tr}\left((\boldsymbol{\Phi}_S(k) - \tau\boldsymbol{\Phi}_N(k))\mathcal{R}\right) \ge \sigma_d^2\tau, \ k = 0,1,\ldots,N-1\\
& \text{tr}\left(\boldsymbol{\Phi}_P\mathcal{R}\right) \le P_{r,\max}\\
& \mathcal{R} \succeq 0,
\end{aligned} \tag{31}$$

where $\boldsymbol{\Phi}_P = \mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H$, $\boldsymbol{\Phi}_S(k) = NP_{s,k}\sigma_g^2\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H$, and $\boldsymbol{\Phi}_N(k) = \sigma_r^2\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H$. Applying semi-definite relaxation (SDR) to the above problem, we drop the rank constraint $\text{rank}(\mathcal{R}) = 1$ to make the relaxed optimization problem a quasi-convex problem. Then, the solution of the relaxed quasi-convex optimization problem can be obtained efficiently by solving its corresponding feasibility problem [10]:

$$\begin{aligned}
\text{Find} \quad & \mathcal{R}\\
\text{s.t.} \quad & \text{tr}\left((\boldsymbol{\Phi}_S(k) - \tau\boldsymbol{\Phi}_N(k))\mathcal{R}\right) \ge \sigma_d^2\tau, \ k = 0,1,\ldots,N-1\\
& \text{tr}\left(\boldsymbol{\Phi}_P\mathcal{R}\right) \le P_{r,\max}\\
& \mathcal{R} \succeq 0.
\end{aligned} \tag{32}$$

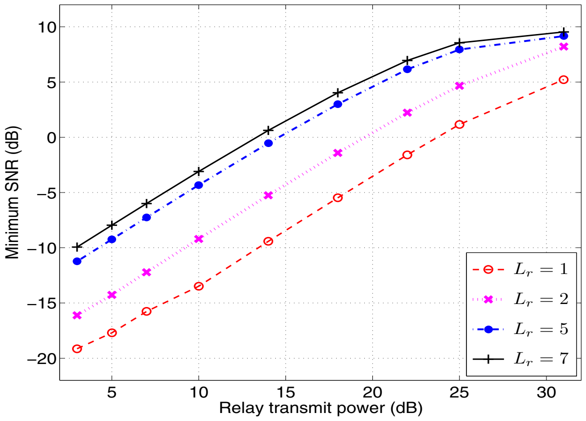

Fig. 1. The worst subcarrier SNR versus relay transmit power

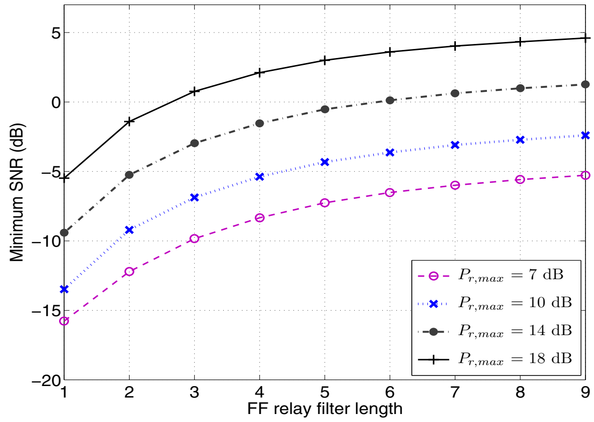

Fig. 2. The worst subcarrier SNR versus relay filter length

Note that the feasible set in (31) is convex for any τ. Suppose that $\tau^\star$ is the optimal value of (31). Then, the feasibility problem (32) is feasible for $\tau \leq \tau^\star$, whereas it is not feasible for $\tau > \tau^\star$. Based on this fact, we can find the solution to (31) by using a simple bisection algorithm, which is given below.

Algorithm 1

Step 1: Select a proper interval s.t. $\tau^\star \in (\tau_l, \tau_r)$.
Step 2: Set $\tau = (\tau_l + \tau_r)/2$.
Step 3: Solve the feasibility problem (32) for τ.
If feasible, $\tau_l = \tau$. Otherwise, $\tau_r = \tau$.
Step 4: Repeat Steps 1 to 3 until $(\tau_r - \tau_l) < \epsilon$.

Here, ϵ is the allowed error tolerance. Note that the feasibility problem (32) is a simple SDP problem, which can be solved efficiently by the interior point method [11]. Due to the relaxation, it is not guaranteed that the algorithm yields the desired rank-one solution. When the result of the algorithm has rank larger than one, randomization techniques [12] can be applied to yield a rank-one solution $\mathbf{r}$ from $\mathcal{R}$.

IV. Numerical results

In this section, we provide some simulation results to examine the performance of the proposed FF relay. We considered a relay network with an OFDM transmitter, an FF relay, and a destination, as described in Section II. The simulation parameters were as follows: $N = 32$, $L_f = L_g = 3$, $f_l \overset{i.i.d.}{\sim} \mathcal{CN}(0,1)$, $g_l \overset{i.i.d.}{\sim} \mathcal{CN}(0,1)$, $\sigma_r^2 = \sigma_d^2 = 1$, and the source transmit power was 20dB higher than the noise power. Figures 1 and 2 show the worst subcarrier SNR versus the relay transmit power and the worst subcarrier SNR versus the relay filter length, respectively. As shown in Figures 1 and 2, the performance gain of the FF relay over the AF relay is significant and most of the gain is achieved only with a few taps in the FIR filter in the FF relay.

V. Conclusion

In this paper, we have considered the FF relay design for OFDM systems for transparent relay operation under the design criterion of maximizing the worst subcarrier SNR subject to the total relay transmit power constraint. We have shown that the design problem becomes a tractable SDP problem and that the FF relay outperforms the AF relay significantly. Thus, the proposed FF relaying scheme provides a practical alternative to the AF relaying scheme for OFDM systems.

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