Filter-and-Forward Transparent Relay Design for OFDM Systems

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***Abstract*—In this paper, the filter-and-forward (FF) relay design for orthogonal frequency-division multiplexing (OFDM) transmission systems is considered for improving system performance over simple amplify-and-forward (AF) relaying. Unlike conventional OFDM relays performing OFDM demodulation and remodulation, to reduce processing complexity, the proposed FF relay directly filters the incoming signal in the time domain with a finite impulse response (FIR) and forwards the filtered signal to the destination. Three design criteria are considered for optimizing the relay filter. The first criterion is the minimization of the relay transmit power subject to per-subcarrier signal-to-noise ratio (SNR) constraints, the second criterion is the maximization of the worst subcarrier channel SNR subject to source and relay transmit power constraints, and the third criterion is the maximization of the data rate subject to source and relay transmit power constraints. It is shown that the first problem reduces to a semi-definite programming (SDP) problem by semi-definite relaxation (SDR), and the solution to the relaxed SDP problem has rank one under a mild condition. For the latter two problems, the problem of joint source power allocation and relay filter design is considered, and an efficient algorithm is proposed for each problem based on alternating optimization and the projected gradient method (PGM). Numerical results show that the proposed FF relay significantly outperforms simple AF relays with an insignificant increase in complexity. Thus, the proposed FF relay provides a practical alternative to the AF relaying scheme for OFDM transmission.**

***Index Terms*—Amplify-and-forward (AF), filter-and-forward (FF), linear relay, orthogonal frequency-division multiplexing (OFDM) systems, semi-definite programming (SDP).**

I. Introduction

RECENTLY, relay networks have drawn extensive interest from the research community because they play an important role in enlarging the network coverage and in improving the system performance in current and future wireless networks. Indeed, Long Term Evolution-Advanced adopts relays for coverage extension and performance improvement [2]. There are several well-known relaying schemes, such as amplify-and-forward (AF), decode-and-forward, and compress-and-forward schemes [3]–[5]. Among the relaying schemes, the AF scheme (i.e., a simple repeater) is the simplest and is suitable for cheap relay deployment under transparent[1] operation [2]. Recently, there have been some efforts to extend this simple AF scheme to a linear filtering relaying scheme, i.e., a filter-and-forward (FF) scheme, to obtain better performance than the AF scheme while keeping the benefit of low computational complexity of the AF scheme [6]–[11]. It has been shown that the FF scheme can considerably outperform the AF scheme. However, most of the previous works on the FF relay have been done for single-carrier transmission, whereas most of the current wireless standards adopt orthogonal frequency-division multiplexing (OFDM) transmission. Thus, in this paper, we propose direct FF relaying for OFDM transmission, instead of using conventional OFDM relays that OFDM-demodulate the incoming signal, amplify or decode the demodulated signal, OFDM-remodulate the processed signal, and transmit the remodulated OFDM signal to the destination [12]–[15]. In the proposed scheme, the incoming signal to the relay is filtered with a finite impulse response (FIR) at the *chip rate* of the OFDM modulation in the time domain, and the filtered signal is directly forwarded to the destination. In this way, the necessity of OFDM processing at the relay is eliminated, but the overall performance can be still improved over the AF scheme by a properly designed relay filter.

A. Our Approach and Contributions

In this paper, we consider three meaningful criteria for the FF relay design for OFDM systems, i.e., minimization of power consumption at the relay, maximization of the worst subcarrier SNR, and maximization of the data rate, under the scenario of single-input–single-output (SISO) communication as the first step in this research direction. [The multiple-input–multiple-output (MIMO) communication case is beyond the scope of this paper and will be studied as a future work.] Our contributions on this topic are summarized in the following.

- First, by exploiting the eigenproperty of circulant matrices and the structure of Toeplitz filtering matrices, we derived necessary expressions for the problem formulation, such as the subcarrier SNR and the relay transmit power, in terms of the design variables of the relay filter coefficients

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[1]Transparent operation means that the destination node does not know the existence of the relay node, and this operation is suitable for cheap AF relays [2].

and source power allocation, and explicitly formulated the given problems based on the derived expressions.

- In the case of the first problem of relay power minimization under per-subcarrier SNR constraints, we showed that the problem is expressed as a semi-definite relaxation (SDR) problem. That is, the original nonconvex FF relay design problem is approximated by a convex semi-definite programming (SDP) problem. Furthermore, in this case, we showed that the solution to the relaxed SDP problem is the same as that to the original problem under a mild condition.
- For the second design criterion, we formulated the problem of joint design of the relay filter and the source power allocation for the worst subcarrier SNR maximization subject to source and relay power constraints, and provided an efficient iterative algorithm to solve this problem based on alternating optimization. The provided algorithm consists of two steps at each iteration, i.e., optimizing the relay filter to maximize the worst subcarrier SNR for a given power allocation and optimizing the source power allocation to maximize the worst subcarrier SNR for a given relay filter, and guarantees convergence to a locally optimal point, although the convergence to a globally optimal point is not guaranteed. We showed that the first step of the iteration reduces to an SDP problem, and the second step of the iteration reduces to linear programming (LP). The second criterion is closely related to bit-error-rate (BER) minimization in the case of weak or no channel coding in addition to overall quality-of-service (QoS) improvement for subcarrier channels. This is because, in the single-user case, bits for one user are distributed across subcarriers, and the overall system BER is dominated by the BER of the worst subcarrier channel [16].
- For the third problem of joint optimization of the relay filter and the source power allocation for rate maximization, we proposed an efficient algorithm by directly applying the projected gradient method (PGM) [11], [17]–[19] to this constrained optimization problem. The proposed method guarantees the satisfaction of the constraints and convergence to a locally optimal point.

Numerical results show that the proposed FF relay significantly outperforms simple AF relays and furthermore achieves most of the performance gain with a few filter taps. Thus, the proposed FF relay provides a practical alternative with low complexity to the AF relaying scheme for OFDM transmission.

B. Notation and Organization

In this paper, we will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For matrix $\mathbf{A}$, $\mathbf{A}^*$, $\mathbf{A}^T$, $\mathbf{A}^H$, and $\mathrm{tr}(\mathbf{A})$ indicate the complex conjugate, transpose, conjugate transpose, and trace of $\mathbf{A}$, respectively. $\mathbf{A} \succeq 0$ and $\mathbf{A} \succ 0$ mean that $\mathbf{A}$ is positive semi-definite and that $\mathbf{A}$ is strictly positive definite, respectively. For two matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \succeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B} \succeq 0$. $\mathbf{I}_n$ stands for the identity matrix of size n (the subscript is omitted when unnecessary), and $\mathbf{0}_{m\times n}$ denotes a $m \times n$ matrix with all zero elements. The notation Toeplitz$(\mathbf{f}^T, N)$ indicates a $N \times (N + L_f - 1)$ Toeplitz matrix with N rows and $[\mathbf{f}^T, 0, \dots, 0]$ as its first row vector, where $\mathbf{f}^T$ is a row vector of size L_f, and $\mathrm{diag}(d_1, \dots, d_n)$ means a diagonal matrix with diagonal elements $d_1, \dots, d_n$. Notation $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is complex circularly symmetric Gaussian distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\mathbb{E}\{\cdot\}$ denotes the expectation. $j = \sqrt{-1}$.

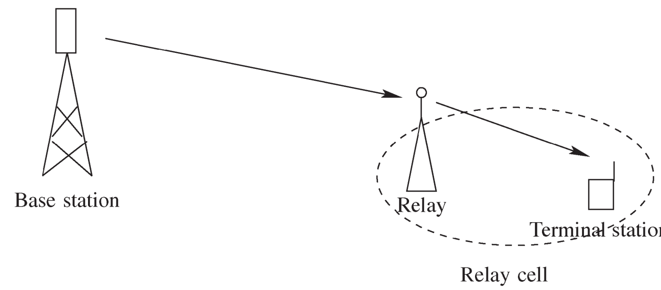


Fig. 1. Considered relay network.

The remainder of this paper is organized as follows. The system model is described in Section II. In Section III, the FF relay design problems are formulated and solved. In Section IV, the performance of the proposed design methods is investigated. Several issues regarding practical implementation of the proposed FF relay are discussed in Section V, which is followed by the conclusion in Section VI.

II. System Model

We consider a full-duplex[2] relay network composed of a source node (base station), a relay node, and a destination node (terminal station), as shown in Figs. 1 and 2, where the source employs OFDM modulation with N subcarriers, and each link performs SISO communication. We consider the case that the direct link between the source and the destination is seriously faded. Thus, for simplicity, we assume that there is no direct link between the source and the destination and that both the source-to-relay (SR) link and the relay-to-destination (RD) link are frequency-selective channels modeled as multitap filters with FIRs. We assume that the relay is an FF relay, i.e., the relay performs FIR filtering on the incoming signal at the chip rate of the OFDM modulation and immediately transmits the filtered output to the destination.[3] Thus, the FF relay can be regarded as an additional frequency-selective (time dispersive) channel between the source and the destination. We assume that the order of the FIR filter at the relay does not make the length of the overall FIR channel between the source and the destination larger than that of the OFDM cyclic prefix. Since our focus of using the FF relay in this paper is the transparent relay operation, we assume that the SR channel state is known to the relay and that the RD channel state is unknown to the relay, but the RD channel distribution is known to the relay. This assumption is reasonable for the transparent relay operation since the relay node does not have its own identity, and the

[2]See Section V.

[3]This FF scheme requires up- and downconverters and simple baseband circuitry only.

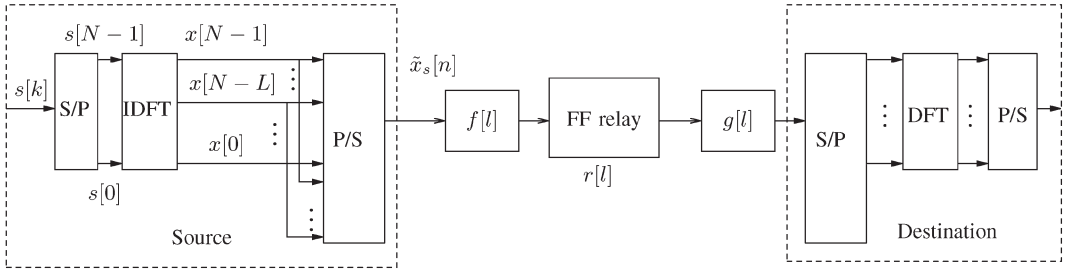


Fig. 2. Considered relay network with an OFDM transmitter, an FF relay, and a destination node.

destination node does not know the existence of the relay; thus, the destination node does not feed back information to the relay node directly. (See Section V regarding how to obtain channel information.)

Specifically, at the source, the length N data vector of OFDM symbols is given by $\mathbf{s} := [s[N-1], s[N-2], \ldots, s[0]]^T$, where each data symbol is assumed to be a zero-mean independent complex Gaussian random variable with variance $P_{s,k}$, i.e., $s[k] \sim \mathcal{CN}(0, P_{s,k})$ for $k = 0, 1, \ldots, N-1$. The time-domain signal vector $\mathbf{x}_s = [x_s[N-1], x_s[N-2], \ldots, x_s[0]]^T$ after the normalized inverse discrete Fourier transform (IDFT) at the source is given by

$$\mathbf{x}_s = \mathbf{W}_N \mathbf{s} \tag{1}$$

where

$$\mathbf{W}_N = \frac{1}{\sqrt{N}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & \omega_N & \cdots & \omega_N^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \omega_N^{N-1} & \cdots & \omega_N^{(N-1)^2} \end{bmatrix}$$

and $\omega_N = e^{j(2\pi/N)}$. Let $\mathbf{w}_{k-1}^T$ denote the kth row of the normalized IDFT matrix $\mathbf{W}_N$ for $k = 1, \ldots, N$. Then, $\mathbf{x}_s$ can be written as $\mathbf{x}_s = [\mathbf{w}_0^T \mathbf{s}, \mathbf{w}_1^T \mathbf{s}, \ldots, \mathbf{w}_{N-1}^T \mathbf{s}]^T$, and covariance matrix $\mathbf{\Sigma}_{\mathbf{x}_s} = \mathbb{E}\{\mathbf{x}_s \mathbf{x}_s^H\}$ of $\mathbf{x}_s$ is given by

$$\mathbf{\Sigma}_{\mathbf{x}_s} = \begin{bmatrix} \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} & \frac{1}{\sqrt{N}}\mathbf{p}_s^T \mathbf{w}_1^* & \cdots & \frac{1}{\sqrt{N}}\mathbf{p}_s^T \mathbf{w}_{N-1}^* \\ \frac{1}{\sqrt{N}}\mathbf{w}_1^T \mathbf{p}_s & \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} & \cdots & \frac{1}{\sqrt{N}}\mathbf{p}_s^T \mathbf{w}_{N-2}^* \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{\sqrt{N}}\mathbf{w}_{N-1}^T \mathbf{p}_s & \frac{1}{\sqrt{N}}\mathbf{w}_{N-2}^T \mathbf{p}_s & \cdots & \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k} \end{bmatrix} \tag{2}$$

where $\mathbf{p}_s = [P_{s,N-1}, P_{s,N-2}, \ldots, P_{s,0}]^T$ is the vector composed of source power assigned to each subcarrier, since

$$\mathbb{E}\left\{\mathbf{w}_i^T \mathbf{s}\mathbf{s}^H \mathbf{w}_j^*\right\} = \begin{cases} \frac{1}{\sqrt{N}}\mathbf{w}_{I-j}^T \mathbf{p}_s, & \text{if } i > j \\ \frac{1}{N}\sum_{k=0}^{N-1} P_{s,k}, & \text{if } i = j \\ \frac{1}{\sqrt{N}}\mathbf{p}_s^T \mathbf{w}_{j-i}^*, & \text{if } i < j. \end{cases} \tag{3}$$

Vector $\mathbf{x}_s$ is attached by a cyclic prefix with length $L_{\rm CP}$, i.e.,

$$\tilde{x}_s[n] = \begin{cases} x_s[n], & n = 0, 1, \ldots, N-1 \\ x_s[N+n], & n = -1, -2, \ldots, -L_{\rm CP} \end{cases} \tag{4}$$

and the cyclic-prefix-attached sequence $\tilde{x}_s[n]$ is transmitted from the source to the relay through the SR channel. Then, the received baseband signal at the relay is given by

$$y_r[n] = \sum_{l=0}^{L_f-1} f_l \tilde{x}_s[n-l] + n_r[n] \tag{5}$$

where $\mathbf{f} = [f_0, f_1, \ldots, f_{L_f-1}]^T$ is the channel tap coefficient vector of the SR channel known to the relay, L_f is the length of the SR FIR channel, and $n_r[n]$ is the additive white Gaussian noise at the relay with $n_r[n] \sim \mathcal{CN}(0, \sigma_r^2)$. At the relay, the received signal $y_r[n]$ is *FIR-filtered at the chip rate of the OFDM transmission* and is then immediately transmitted to the destination. Thus, the output signal at the relay at (chip) time n is given by

$$y_t[n] = \sum_{l=0}^{L_r-1} r_l \, y_r[n-l] \tag{6}$$

where $\mathbf{r} = [r_0, r_1, \ldots, r_{L_r-1}]^T$ is the FIR filter coefficient vector at the relay and L_r is the order of the FIR filter. Note that, when $L_r = 1$, the FF relay simply reduces to the AF relay. However, when $L_r > 1$, the FF relay is an extension of the AF relay with some amount of digital processing. Finally, the signal transmitted by the relay goes through the RD FIR channel to the destination. Thus, the received signal at the destination is given by

$$y_d[n] = \sum_{l=0}^{L_g-1} g_l \, y_t[n-l] + n_d[n] \tag{7}$$

where $\mathbf{g} = [g_0, g_1, \ldots, g_{L_g-1}]^T$ is the FIR channel tap coefficient vector for the RD channel, L_g is the order of the RD FIR channel, and $n_d[n]$ is zero-mean white Gaussian noise with variance σ_d^2 at the destination. Here, we assume that the channel tap coefficient g_l, $l = 0, 1, \ldots, L_g - 1$, is independent and identically distributed (i.i.d.) according to $g_l \overset{\text{i.i.d.}}{\sim} \mathcal{CN}(0, \sigma_g^2)$, i.e., each tap is independently Rayleigh faded and that the realization $\{g_l, l = 0, 1, \ldots, L_g - 1\}$ is not known to the relay, but its distribution is known to the relay. By stacking the output symbols at the relay and the received symbols at the destination, we have the following vectors for the transmitted signal at the relay and the cyclic-prefix-portion-removed received signal vector at the destination, respectively:

$$\mathbf{y}_t = \mathbf{R}\mathbf{F}\tilde{\mathbf{x}}_s + \mathbf{R}\mathbf{n}_r \quad \mathbf{y}_d = \tilde{\mathbf{G}}\mathbf{R}\mathbf{F}\tilde{\mathbf{x}}_s + \mathbf{G}\mathbf{R}\mathbf{n}_r + \mathbf{n}_d \tag{8}$$

where

$$\begin{aligned}
\mathbf{y}_d &= [y_d[N-1], y_d[N-2], \ldots, y_d[0]]^T \\
\mathbf{y}_t &= [y_t[N-1], \ldots, y_t[0], \ldots, y_t[-L_g+1]]^T \\
\tilde{\mathbf{x}}_s &= [x_s[N-1], \ldots, x_s[0], \ldots, x_s[-L_g-L_r-L_f+3]]^T \\
\mathbf{n}_r &= [n_r[N-1], \ldots, n_r[0], \ldots, n_r[-L_g-L_r+2]]^T \\
\mathbf{n}_d &= [n_d[N-1], \ldots, n_d[0]]^T \\
\mathbf{G} &= \text{Toeplitz}(\mathbf{g}^T, N) \\
\mathbf{R} &= \text{Toeplitz}\left(\mathbf{r}^T, N+L_g-1\right) \\
\mathbf{F} &= \text{Toeplitz}(\mathbf{f}^T, N+L_g+L_r-2). \qquad (9)
\end{aligned}$$

Under the assumption that $L_{\text{CP}} \geq L_g + L_r + L_f - 3$, the DFT of the cyclic-prefix-portion-removed received vector of size N at the destination is given by

$$\begin{aligned}
\hat{\mathbf{y}}_d &= \mathbf{W}_N^H \mathbf{GRF}\tilde{\mathbf{x}}_s + \mathbf{W}_N^H \mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d \qquad (10) \\
&= \mathbf{W}_N^H \mathbf{H}_c \mathbf{W}_N \mathbf{s} + \mathbf{W}_N^H \mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d \qquad (11) \\
&= \mathbf{D}\mathbf{s} + \mathbf{W}_N^H \mathbf{GR}\mathbf{n}_r + \mathbf{W}_N^H \mathbf{n}_d \qquad (12)
\end{aligned}$$

where $\mathbf{W}_N^H$ is the normalized DFT matrix of size N, $\mathbf{H}_c$ is a $N \times N$ circulant matrix generated from the overall Toeplitz filtering matrix $\mathbf{GRF}$ from the source to the destination, and $\mathbf{D} = \text{diag}(d_0, \ldots, d_{N-1}) = \mathbf{W}_N^H \mathbf{H}_c \mathbf{W}_N$ is the eigendecomposition of $\mathbf{H}_c$.

A. Manipulation for Quadratic Forms

The received signal form (12) is standard in OFDM transmission, but the form cannot be directly used for relay filter optimization in Section III. Thus, here, we derive an explicit expression for the received signal $\hat{y}_d[k]$, $k = 0, 1, \ldots, N-1$, which facilitates optimization formulation in Section III, which is based on the following property of circulant matrices [20].

Lemma 1—[20]: Let $\mathbf{C}$ be an $N \times N$ circulant matrix with the first row $[c(0), c(1), \ldots, c(N-1)]$. Then, the eigenvalues of $\mathbf{C}$ are given by

$$\lambda_k = \sum_{n=0}^{N-1} c(n)\omega_N^{-kn}, \qquad k = 0, 1, \ldots, N-1$$

with the corresponding right eigenvectors

$$\boldsymbol{xi}_k = \frac{1}{\sqrt{N}}\left[1, \omega_N^{-k}, \omega_N^{-2k}, \ldots, \omega_N^{-(N-1)k}\right]^T, \; k = 0, \ldots, N-1.$$

By Lemma 1, to derive the diagonal elements of $\mathbf{D}$ in (12), we need to know only the first row of $\mathbf{H}_c$ in (11). Let the first row of $\mathbf{G}$ be $\tilde{\mathbf{g}}^T$. Then, $\tilde{\mathbf{g}}^T$ is a $1 \times (N + L_g - 1)$ row vector given by

$$\tilde{\mathbf{g}}^T = [\mathbf{g}^T, 0, \ldots, 0] \qquad (13)$$

and the first row of $\mathbf{GRF}$ is given by $\tilde{\mathbf{g}}^T\mathbf{RF}$. Since $\mathbf{H}_c$ is generated by truncating out the elements of $\mathbf{GRF}$ outside the first $N \times N$ positions and by moving the lower $(L_g + L_r + L_f - 3) \times (L_g + L_r + L_f - 3)$ elements of the truncated part to the lower left of the untruncated $N \times N$ matrix, the first row $\mathbf{h}_c^T$ of $\mathbf{H}_c$ is simply the first N elements of the first row $\tilde{\mathbf{g}}^T\mathbf{RF}$ of $\mathbf{GRF}$, i.e., $\mathbf{h}_c^T = \tilde{\mathbf{g}}^T\mathbf{RFT}$, where $\mathbf{T}$ is a truncation matrix for truncating out the elements of $\tilde{\mathbf{g}}^T\mathbf{RF}$, except the first N elements, given by

$$\mathbf{T} = \begin{bmatrix} \mathrm{I}_N \\ \mathbf{0}_{(L_g+L_r+L_f-3)\times N} \end{bmatrix}. \qquad (14)$$

Now, the diagonal elements of $\mathbf{D}$ can be obtained by Lemma 1 and are given by

$$[d_0, \ldots, d_{N-1}]^T = \sqrt{N}\mathbf{W}_N^H(\tilde{\mathbf{g}}^T\mathbf{RFT})^T \qquad (15)$$

where $\sqrt{N}\mathbf{W}_N^H$ is the DFT matrix of size N. Finally, the received signal in the kth subcarrier at the destination is expressed as

$$\hat{y}_d[k] = \sqrt{N}\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}s[k] + \mathbf{w}_k^H\mathbf{GR}\mathbf{n}_r + \mathbf{w}_k^H\mathbf{n}_d \qquad (16)$$

where $\mathbf{w}_k^H$ is the $(k+1)$th row of $\mathbf{W}_N^H$. Thus, the signal and noise parts of $\hat{y}_d[k]$ are given by

$$\begin{aligned}
\hat{y}_{d,S}[k] &= \sqrt{N}\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}s[k] \\
\hat{y}_{d,N}[k] &= \mathbf{w}_k^H\mathbf{GR}\mathbf{n}_r + \mathbf{w}_k^H\mathbf{n}_d \qquad (17)
\end{aligned}$$

respectively, for $k = 0, 1, \ldots, N-1$.

III. Filter-and-Forward Relay Design Criteria and Optimization

Here, we consider three meaningful FF relay design problems for the relay network employing OFDM transmission described in Section II. First, we consider the FF relay filter design to minimize the transmit power of the FF relay subject to an SNR constraint for each OFDM subcarrier channel when the source power allocation $\{P_{s,k}\}$ is given. Here, we shall show that the problem can be formulated as an SDR problem. That is, the original nonconvex FF relay design problem is approximated by a convex SDP problem. Furthermore, in this case, we shall show that the solution to the relaxed SDP problem is the same as that to the original problem under a mild condition. With the formulas for the relay transmit power and the subcarrier channel SNR in terms of the relay filter coefficients and source power allocation obtained to solve the first problem, we next consider two more relay design problems. One is the problem of maximizing the worst subcarrier SNR subject to source and relay power constraints, and the other is the problem of rate maximization subject to source and relay power constraints. The first criterion aims not only at overall QoS improvement for subcarrier channels but also at BER minimization in the case of weak or no channel coding. In the single-user case, bits for one user are distributed across subcarriers, and the overall system BER is dominated by the BER of the worst subcarrier channel since the worst error rate dominates the system error rate [16]. Thus, the maximization of the worst subcarrier SNR is almost equivalent to the minimization of the system BER in the single-user case. For the latter two problems, we consider joint optimization of the relay filter and source power allocation. These two problems are nonconvex optimization problems with respect to the relay filter tap coefficients and the source power allocation. Thus, it is not easy to find a globally optimal solution. To circumvent this difficulty, we apply alternating optimization and the PGM to the worst subcarrier SNR maximization problem and the rate maximization problem, respectively, and propose an efficient iterative algorithm for each problem that converges to a locally optimal point at least.

A. FF Relay Transmit Power Minimization Under Per-Subcarrier SNR Constraints

We first consider the problem of designing the FF relay tap coefficient vector $\mathbf{r} = [r_0, r_1, \ldots, r_{L_r-1}]^T$ to minimize the relay transmit power subject to an SNR constraint per OFDM subcarrier. This problem is formulated as follows.

Problem 1: For given source power allocation $\{P_{s,k},\ k = 0, 1, \ldots, N-1\}$, SR channel $\mathbf{f}$, RD channel static information (σ_g^2, L_g), FF relay filter order L_r, and a set $\{\gamma_k,\ k \in \mathcal{I}\}$ of desired minimum SNR values for subcarrier channels $\mathcal{I}$

$$\min_{\mathbf{r}} \ P_r \ \text{ s.t. } \ \text{SNR}_k \ge \gamma_k, \quad \forall\, k \in \mathcal{I} \subset \{0, 1, \ldots, N-1\} \tag{18}$$

where P_r is the relay transmit power, and SNR_k is the SNR of the kth subcarrier channel.

To solve Problem 1, we need to express each term in the problem as a function of the design variable $\mathbf{r}$. First, let us derive the SNR on the kth subcarrier channel in the received signal (16) at the destination. Note that the signal and noise parts in (17) are represented in terms of the relay filtering matrix $\mathbf{R}$. The representation of SNR in terms of $\mathbf{R}$ is redundant since the true variable $\mathbf{r}$ is embedded in $\mathbf{R}$. Thus, we need reparameterization of the SNR in terms of $\mathbf{r}$, and this can be done based on (17) by exploiting the Toeplitz structure of $\mathbf{R}$ as follows. Using (17), we first express the received signal power at the destination in terms of $\mathbf{r}$ as

$$\begin{aligned}
&\mathbb{E}\left\{|\hat{y}_{d,S}[k]|^2\right\} \\
&= N\mathbb{E}\left\{\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{g}}\,|s[k]|^2\,\tilde{\mathbf{g}}^H\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\right\} \\
&\overset{(a)}{=} NP_{s,k}\text{tr}\left(\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\mathbb{E}\left\{\tilde{\mathbf{g}}\tilde{\mathbf{g}}^H\right\}\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\right) \\
&\overset{(b)}{=} NP_{s,k}\sigma_g^2\text{tr}\left(\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T\mathbf{R}^T\tilde{\mathbf{I}}_{L_g}\mathbf{R}^*\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\right) \\
&\overset{(c)}{=} NP_{s,k}\sigma_g^2\text{tr}\left(\underbrace{\mathbf{F}^*\mathbf{T}^*\mathbf{w}_k\mathbf{w}_k^H\mathbf{T}^T\mathbf{F}^T}_{=:\mathbf{K}_k}\mathbf{R}_{L_g}^T\mathbf{R}_{L_g}^*\right) \\
&\overset{(d)}{=} NP_{s,k}\sigma_g^2\text{tr}\left(\mathbf{R}_{L_g}^*\mathbf{K}_k\mathbf{R}_{L_g}^T\right) \\
&\overset{(e)}{=} NP_{s,k}\sigma_g^2\left[\text{vec}\left(\mathbf{R}_{L_g}^T\right)\right]^H\bar{\mathbf{K}}_k\text{vec}\left(\mathbf{R}_{L_g}^T\right) \\
&= NP_{s,k}\sigma_g^2\left[\text{vec}(\mathbf{R}^T)\right]^H\tilde{\mathbf{K}}_k\ \text{vec}(\mathbf{R}^T) \\
&\overset{(f)}{=} NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}
\end{aligned} \tag{19}$$

$$\mathbf{E}_1 = \Big[\underbrace{\mathbf{I}_{L_r}, \mathbf{0}_{L_r\times(N+L_g-2)}}_{N+L_g+L_r-2 \text{ columns}}, \underbrace{\mathbf{0}_{L_r\times 1}, \mathbf{I}_{L_r}, \mathbf{0}_{L_r\times(N+L_g-3)}}_{N+L_g+L_r-2 \text{ columns}}, \ldots, \underbrace{\mathbf{0}_{L_r\times(N+L_g-2)}, \mathbf{I}_{L_r}}_{N+L_g+L_r-2 \text{ columns}}\Big] \tag{20}$$

where

$$\begin{aligned}
\tilde{\mathbf{I}}_{L_g} &:= \begin{bmatrix} \mathbf{I}_{L_g} & \mathbf{0}_{L_g\times(N-1)} \\ \mathbf{0}_{(N-1)\times L_g} & \mathbf{0}_{(N-1)\times(N-1)} \end{bmatrix} \\
\mathbf{R} &= \begin{bmatrix} \mathbf{R}_{L_g} \\ \mathbf{R}_{N-1} \end{bmatrix} \\
\bar{\mathbf{K}}_k &= \mathbf{I}_{L_g} \otimes \mathbf{K}_k \\
\tilde{\mathbf{K}}_k &= \tilde{\mathbf{I}}_{L_g} \otimes \mathbf{K}_k
\end{aligned}$$

and $\mathbf{R}_{L_g}$ is a matrix composed of the first L_g rows of $\mathbf{R}$.

Here, (a) holds due to the assumption of independence of the signal and the RD channel coefficients, (b) holds due to the assumption[4] of $g_l \overset{\text{i.i.d.}}{\sim} \mathcal{CN}(0, \sigma_g^2)$ [see (13)], (c) and (d) hold due to $\text{tr}(\mathbf{ABC}) = \text{tr}(\mathbf{CAB})$, (e) holds due to $\text{tr}(\mathbf{R}_{L_g}^*\mathbf{K}_k\mathbf{R}_{L_g}^T) = [\text{vec}(\mathbf{R}_{L_g}^T)]^H\bar{\mathbf{K}}_k\text{vec}(\mathbf{R}_{L_g}^T)$, and (f) is obtained because $\mathbf{R} = \text{Toeplitz}(\mathbf{r}^T, N + L_g - 1)$, and thus, $\text{vec}(\mathbf{R}^T) = \mathbf{E}_1^H\mathbf{r}$. The key point of the derivation of (19) is that the received signal power at the kth subcarrier channel is represented as a quadratic form of the design variable $\mathbf{r}$. Next, consider the received noise power for the kth subcarrier channel. Using similar techniques to those used to obtain (19), we can express the received noise power based on the noise part in (17) as

$$\begin{aligned}
\mathbb{E}\left\{|\hat{y}_{d,N}[k]|^2\right\} &= \sigma_r^2\text{tr}\left(\mathbf{R}^H\underbrace{\mathbb{E}\left\{\mathbf{G}^H\mathbf{w}_k\mathbf{w}_k^H\mathbf{G}\right\}}_{=:\mathbf{M}_k}\mathbf{R}\right) + \sigma_d^2 \\
&= \sigma_r^2\left[\text{vec}(\mathbf{R})\right]^H\tilde{\mathbf{M}}_k\text{vec}(\mathbf{R}) + \sigma_d^2 \\
&= \sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r} + \sigma_d^2
\end{aligned} \tag{21}$$

where $\tilde{\mathbf{M}}_k = \mathbf{I}_{N+L_g+L_r-2} \otimes \mathbf{M}_k$, and $\mathbf{E}_2$ is given by

$$\mathbf{E}_2 = \begin{bmatrix}
\mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T & \mathbf{0}^T & \cdots & \mathbf{0}^T \\
\mathbf{0}^T & \mathbf{e}_1^T & \mathbf{e}_2^T & \cdots & \mathbf{e}_{N+L_g-1}^T & \cdots & \mathbf{0}^T \\
\mathbf{0}^T & \mathbf{0}^T & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & & & \ddots & \ddots & \ddots & \mathbf{0}^T \\
\mathbf{0}^T & \cdots & \cdots & \mathbf{0}^T & \mathbf{e}_1^T & \cdots & \mathbf{e}_{N+L_g-1}^T
\end{bmatrix}. \tag{22}$$

Here, $\mathbf{e}_i^T$ is the ith row of $\mathbf{I}_{N+L_g-1}$, and the size of each $\mathbf{0}^T$ in (22) is $1 \times (N + L_g - 1)$. (It is easy to verify that $\text{vec}(\mathbf{R}) =$

[4]The i.i.d. assumption for g_l is not necessary. More general cases, such as correlated g_l and deterministic g_l, can be included in the proposed framework with a slight change in the derivation.

$\mathbf{E}_2^H\mathbf{r}$ due to the Toeplitz structure of $\mathbf{R}$.) Based on (19) and (21), the SNR of the kth subcarrier channel is expressed as

$$\mathrm{SNR}_k = \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}. \quad (23)$$

Next, consider the relay transmit power. Using (8), we obtain the relay transmit power in a similar way as

$$\begin{aligned}\mathbb{E}\left\{\mathrm{tr}\left(\mathbf{y}_t\mathbf{y}_t^H\right)\right\} &= \mathrm{tr}\left(\mathbf{R}\mathbf{F}\underbrace{\mathbb{E}\left\{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s^H\right\}}_{=:\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}_s}}\mathbf{F}^H\mathbf{R}^H\right)+\mathrm{tr}\left(\sigma_r^2\mathbf{R}\mathbf{R}^H\right)\\ &= \mathrm{tr}\left(\mathbf{R}\underbrace{\left(\mathbf{F}\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}_s}\mathbf{F}^H+\sigma_r^2\mathbf{I}\right)}_{=:\boldsymbol{\Pi}}\mathbf{R}^H\right)\\ &= \left[\mathrm{vec}(\mathbf{R}^H)\right]^H\tilde{\boldsymbol{\Pi}}\mathrm{vec}(\mathbf{R}^H)\\ &= \mathbf{r}^T\mathbf{E}_1\tilde{\boldsymbol{\Pi}}\mathbf{E}_1^H\mathbf{r}^* = \mathbf{r}^H\mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \end{aligned} \quad (24)$$

where $\tilde{\boldsymbol{\Pi}} = \mathbf{I}_{N+L_g-1}\otimes\boldsymbol{\Pi}$, and $\boldsymbol{\Sigma}_{\tilde{\mathbf{x}}_s}$ is obtained similarly to (2) based on (3). The last equality holds since the power is a real-valued quantity.

Now, based on (19), (21), and (24), Problem 1 can be restated as follows:

$$\begin{aligned}&\min_{\mathbf{r}}\ \mathbf{r}^H\mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H\mathbf{r}\\ &\text{s.t.}\ \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}\geq\gamma_k,\quad k\in\mathcal{I}.\end{aligned} \quad (25)$$

The given problem is not a convex problem. However, the problem can be still efficiently solved by using convex optimization techniques. Let $\mathcal{R} := \mathbf{r}\mathbf{r}^H$. Then, by using $\mathrm{tr}(\mathbf{ABC}) = \mathrm{tr}(\mathbf{BCA})$ and by relaxing the rank-one constraint for $\mathcal{R}$, problem (25) can be reformulated as follows.

Problem 1′:

$$\begin{aligned}&\min_{\mathcal{R}}\quad \mathrm{tr}(\boldsymbol{\Phi}_P\mathcal{R})\\ &\text{s.t.}\quad \mathrm{tr}\left(\left[\boldsymbol{\Phi}_S(k)-\gamma_k\boldsymbol{\Phi}_N(k)\right]\mathcal{R}\right)\geq\sigma_d^2\gamma_k,\quad k\in\mathcal{I},\\ &\qquad\ \mathcal{R}\succeq 0\end{aligned} \quad (26)$$

where $\boldsymbol{\Phi}_P = \mathbf{E}_1\tilde{\boldsymbol{\Pi}}^*\mathbf{E}_1^H$, $\boldsymbol{\Phi}_S(k) = NP_{s,k}\sigma_g^2\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H$, and $\boldsymbol{\Phi}_N(k) = \sigma_r^2\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H$.

Note that by relaxing the rank-one constraint for $\mathcal{R}$, Problem 1 is converted to Problem 1′, which is an SDP [21], and it can be efficiently solved by using the standard interior point method for convex optimization [21], [22]. With an additional constraint $\mathrm{rank}(\mathcal{R}) = 1$, Problem 1′ is equivalent to the original Problem 1. That is, if the optimal solution to Problem 1′ has rank one, then it is also the optimal solution to Problem 1. However, there is no guarantee that an algorithm for solving Problem 1′ yields the desired rank-one solution. In such a case, randomization techniques [23] can be used to obtain a rank-one solution $\mathbf{r}$ from $\mathcal{R}$. However, for this specific problem related to the transparent FF relay design, we provide a stronger result, which is stated in the following theorem.

Theorem 1: If all the desired SNR constraints except one are satisfied with strict inequality, then the nontrivial optimal solution of Problem 1′, which is a relaxed version of Problem 1, always has rank one.

Proof: See the Appendix.

Note that the condition in Theorem 1 is mild and is satisfied in many cases. Thus, solving Problem 1′ directly yields the solution to the original power minimization problem under subcarrier SNR constraints in many cases.

B. Worst Subcarrier SNR Maximization

Now, we consider the second FF relay design problem of maximizing the SNR of the worst subcarrier channel under transmit power constraints. As aforementioned, this problem is closely related to BER minimization in the case of weak or no channel coding in addition to minimum QoS improvement. To be complete, for this important problem, we consider not only relay filter optimization but also optimal source power allocation. The joint optimization yields a further gain over the relay filter optimization only, as seen in other joint optimization [13], [24], [25]. The problem of joint source power allocation and FF relay filter design to maximize the worst subcarrier SNR subject to total source and relay transmit power constraints is formulated as follows.

Problem 2: For a given SR channel $\mathbf{f}$, RD channel statistic information (σ_g^2, L_g), FF relay filter order L_r, maximum available source transmit power $P_{s,\max}$, and maximum available relay transmit power $P_{r,\max}$, optimize the relay filter $\mathbf{r}$ and the source power allocation $\{P_{s,0},\ldots,P_{s,N-1}\}$ to maximize the worst subcarrier SNR as follows:

$$\begin{aligned}&\max_{\mathbf{r},P_{s,0},\ldots,P_{s,N-1}}\ \min_{k\in\{0,\ldots,N-1\}}\ \mathrm{SNR}_k\\ &\text{s.t.}\quad \sum_{k=0}^{N-1}P_{s,k}\leq P_{s,\max},\ P_r\leq P_{r,\max}.\end{aligned} \quad (27)$$

Note that Problem 2 is a complicated nonconvex optimization problem. There exist several methods that can find the optimal solution of a nonconvex optimization problem as long as the cost function is not too complicated [26], [27]. However, these methods require high computational complexity. Hence, here, we approach the problem by using a suboptimal alternating optimization technique for computational efficiency. That is, first, the source power allocation is properly initialized, and Problem 2 is solved to optimize the relay filter for a given source power allocation. (This problem is defined as Problem 2-1.) Then, with the given relay filter tap coefficients obtained by solving Problem 2-1, the source power allocation is optimized. (This problem is defined as Problem 2-2.) The two problems are solved in an alternating fashion until the iteration converges. Let us consider Problem 2-1 first. Problem 2-1 can be explicitly written based on (23) and (24) as follows.

Problem 2-1: For given source power allocation $\{P_{s,k}, k = 0,1,\ldots,N-1\}$, SR channel $\mathbf{f}$, RD channel static information

(σ_g^2, L_g), FF relay filter order L_r, and maximum available relay transmit power $P_{r,\max}$

$$\max_{\mathbf{r}} \quad \min_{k\in\{0,\ldots,N-1\}} \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2}$$
$$\text{s.t.} \quad \mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \le P_{r,\max}. \tag{28}$$

By introducing slack variable τ, the given max–min problem can be rewritten as

$$\max_{\mathbf{r}} \quad \tau$$
$$\text{s.t.} \quad \frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2} \ge \tau,\ k=0,1,\ldots,N-1,$$
$$\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \le P_{r,\max}. \tag{29}$$

Note that this is a nonconvex problem. Again, as in earlier, we convert the problem to a tractable convex problem by SDR as follows:

$$\max_{\mathcal{R}} \quad \tau$$
$$\text{s.t.} \quad \mathrm{tr}\left(\left(\mathbf{\Phi}_S(k)-\tau\mathbf{\Phi}_N(k)\right)\mathcal{R}\right) \ge \sigma_d^2\tau,\ k=0,\ldots,N-1,$$
$$\mathrm{tr}\left(\mathbf{\Phi}_P\mathcal{R}\right) \le P_{r,\max}, \quad \mathcal{R}\succeq 0 \tag{30}$$

where $\mathbf{\Phi}_S(k)$, $\mathbf{\Phi}_N(k)$, and $\mathbf{\Phi}_P$ are already defined in Problem $1'$. In problem (30), the rank constraint $\mathrm{rank}(\mathcal{R})=1$ is dropped by SDR as in Problem $1'$. Note that the relaxed optimization problem is quasi-convex, i.e., for given τ, the problem is convex. The solution of the quasi-convex optimization problem can be obtained by solving its corresponding feasibility problem [28]

$$\text{Find} \quad \mathcal{R}$$
$$\text{s.t.} \quad \mathrm{tr}\left(\left(\mathbf{\Phi}_S(k)-\tau\mathbf{\Phi}_N(k)\right)\mathcal{R}\right) \ge \sigma_d^2\tau,\ k=0,\ldots,N-1,$$
$$\mathrm{tr}(\mathbf{\Phi}_P\mathcal{R}) \le P_{r,\max}, \quad \mathcal{R}\succeq 0. \tag{31}$$

The feasible set in problem (30) is convex for any value of τ. Let $\tau^\star$ be the optimal value of problem (30). Then, we can find the solution to problem (30) by using the fact that the feasibility problem (31) is feasible for $\tau \le \tau^\star$, whereas it is not feasible for $\tau > \tau^\star$. Thus, we propose a simple bisection algorithm to solve problem (30), which is a relaxed version of Problem 2-1, as follows.

Algorithm 1: Choose some appropriate interval subject to $\tau^\star \in (\tau_L, \tau_R)$.

Step 1: Set $\tau = (\tau_L+\tau_R)/2$.

Step 2: Solve the feasibility problem (31) for τ. If it is feasible, $\tau_L=\tau$. Otherwise, $\tau_R=\tau$.

Step 3: Repeat Steps 1 and 2 until $(\tau_R-\tau_L)<\epsilon$.

Here, ϵ is the allowed error tolerance for τ. Note that the given feasibility problem is a standard SDP problem, which can be easily solved by the interior point method [22]. Due to the relaxation, matrix $\mathcal{R}$ obtained by solving the relaxed optimization problem may not have rank one in general. In this case, randomization techniques can be applied to find a rank-one solution.

When we optimize only the relay filter to maximize the worst subcarrier SNR for a given source power allocation, we can simply use Algorithm 1 only. However, for joint optimization of the relay filter and the source power allocation by alternating optimization, we need to consider Problem 2-2, which is given as follows.

Problem 2-2: For given FF relay filter $\mathbf{r}$, SR channel $\mathbf{f}$, RD channel statistic information (σ_g^2, L_g), maximum allowed source transmit power $P_{s,\max}$, and maximum allowed relay transmit power $P_{r,\max}$

$$\max_{P_{s,0},\ldots,P_{s,N-1}} \quad \min_{k\in\{0,\ldots,N-1\}} \mathrm{SNR}_k$$
$$\text{s.t.} \quad \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max},\ P_r \le P_{r,\max},\ \mathrm{SNR}_k \ge \tau_0,\ \forall k \tag{32}$$

where τ_0 is the allowed minimum for the worst subcarrier SNR.

Constraint $\mathrm{SNR}_k \ge \tau_0 \forall k$ in (32) is intentionally introduced to guarantee that the proposed alternating algorithm yields a monotone nondecreasing sequence of the worst subcarrier SNR values. (This will become clear shortly.) By introducing slack variable τ and using (23) and (24), Problem 2-2 can be rewritten as follows:

$$\max_{P_{s,0},\ldots,P_{s,N-1},\tau} \quad \tau \tag{33}$$
$$\text{s.t.} \quad \mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{\Pi}}^*\mathbf{E}_1^H\mathbf{r} \le P_{r,\max},$$
$$\frac{NP_{s,k}\sigma_g^2\mathbf{r}^H\mathbf{E}_1\tilde{\mathbf{K}}_k\mathbf{E}_1^H\mathbf{r}}{\sigma_r^2\mathbf{r}^H\mathbf{E}_2\tilde{\mathbf{M}}_k\mathbf{E}_2^H\mathbf{r}+\sigma_d^2} \ge \tau,\ \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max},$$
$$\tau \ge \tau_0,\ k=0,1,\ldots,N-1. \tag{34}$$

Note that, without the relay power constraint in (34), the given problem is a simple LP with respect to $P_{s,0},\ldots,P_{s,N-1}$ and τ. Indeed, the problem is an LP since the relay power constraint can be also written as a linear form in terms of $P_{s,k}$, as shown in the following. The relay power (24) can be rewritten as

$$\begin{aligned}
&\mathbb{E}\left\{\mathrm{tr}(\mathbf{y}_t\mathbf{y}_t^H)\right\}\\
&= \mathrm{tr}\left(\mathbf{RF}\mathbb{E}\left\{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s^H\right\}\mathbf{F}^H\mathbf{R}^H\right)+\mathrm{tr}\left(\sigma_r^2\mathbf{RR}^H\right)\\
&= \mathrm{tr}\left(\mathbb{E}\left\{\tilde{\mathbf{x}}_s\tilde{\mathbf{x}}_s^H\right\}\mathbf{F}^H\mathbf{R}^H\mathbf{RF}\right)+\mathrm{tr}\left(\sigma_r^2\mathbf{RR}^H\right)\\
&\overset{(a)}{=} \mathrm{tr}\left(\tilde{\mathbf{W}}_N\mathbb{E}\{\mathbf{ss}^H\}\tilde{\mathbf{W}}_N^H\mathbf{F}^H\mathbf{R}^H\mathbf{RF}\right)+\mathrm{tr}\left(\sigma_r^2\mathbf{RR}^H\right)\\
&\overset{(b)}{=} \sum_{k=0}^{N-1} P_{s,k}\,\mathrm{tr}\left(\mathbf{e}_{k+1}\mathbf{e}_{k+1}^T\tilde{\mathbf{W}}_N^H\mathbf{F}^H\mathbf{R}^H\mathbf{RF}\tilde{\mathbf{W}}_N\right)\\
&\quad +\mathrm{tr}\left(\sigma_r^2\mathbf{RR}^H\right)
\end{aligned} \tag{35}$$

where $\mathbf{e}_k$ is defined in (22), and $\tilde{\mathbf{W}}_N$ is the cyclic-prefix-extended IDFT matrix given by

$$\tilde{\mathbf{W}}_N = [\mathbf{w}_{N-1},\ \mathbf{w}_{N-2},\ldots,\mathbf{w}_{N-L_g-L_r-L_f+3}]^T.$$

Here, (a) can be verified by using (9), and (b) is due to the assumption of $s[k] \sim \mathcal{CN}(0, P_{s,k})$ for $k = 0, 1, \ldots, N-1$. Using the new expression (35) for the relay transmit power, we obtain an LP optimization problem for the source power allocation from problem (33) as

$$\max_{P_{s,0},\cdots,P_{s,N-1},\tau} \quad \tau \tag{36}$$

$$\text{s.t.} \quad \sum_{k=0}^{N-1} P_{s,k} C_1(k) + C_2 \le P_{r,\max},$$
$$P_{s,k} C_3(k) \ge \tau, \ k = 0, 1, \ldots, N-1,$$
$$\sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max}, \ \tau \ge \tau_0 \tag{37}$$

where $C_1(k) = \mathrm{tr}(\mathbf{e}_{k+1}\mathbf{e}_{k+1}^T \tilde{\mathbf{W}}_N^H \mathbf{F}^H \mathbf{R}^H \mathbf{R} \mathbf{F} \tilde{\mathbf{W}}_N)$, $C_2 = \mathrm{tr}(\sigma_r^2 \mathbf{R}\mathbf{R}^H)$, and $C_3(k) = N\sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r} / (\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2)$. Since problem (36) is an LP problem, Problem 2-2 can be easily solved by a standard convex optimization solver.

Now, combining Problems 2-1 and 2-2, we present our alternating optimization algorithm for the joint source power allocation and relay filter design problem to maximize the worst subcarrier SNR given in Algorithm 2.

Algorithm 2: Given parameters: $\mathbf{f}$, (σ_g^2, L_g), L_r, $P_{s,\max}$, and $P_{r,\max}$.

Step 1: Initialize $P_{s,k}$ for $k = 0, 1, \ldots, N-1$. For example, $P_{s,k} = P_{s,\max}/N$.

Step 2: Solve Problem 2-1 with Algorithm 1.

Step 3: Set the allowed minimum τ_0 for the worst subcarrier SNR in Problem 2-2 as the maximum value $\tau^\star$ obtained from Algorithm 1 in Step 2.

Step 4: For given $\mathbf{r}$ and τ_0 from Steps 2 and 3, solve Problem 2-2 by solving problem (36) to obtain new $P_{s,k}$, $k = 0, \ldots, N-1$.

Step 5: Go to Step 2. Here, set τ_L of Problem 2-1 as the solution to Problem (36) in Step 4.

Step 6: Repeat Steps 2 to 5 until $|\tau_0 - \tau_L| < \epsilon$.

Here, ϵ is the allowed error tolerance for τ. Note that, at each iteration, the value $\tau^\star$ of the worst subcarrier SNR is monotone nondecreasing. This is because the maximum value of the previous step is set as a lower bound of the current step, and the problem at the current step is feasible since the previous combination of $\{P_{s,k}\}$ and $\mathbf{r}$ achieves the current lower bound. Since $\tau^\star$ is monotone nondecreasing and upper bounded because of finite transmit power $P_{s,\max}$ and $P_{r,\max}$, the proposed algorithm converges to a locally optimal point by the monotone convergence theorem. Although convergence to the global optimum is not guaranteed, it is shown in Section IV that the proposed joint design approach significantly improves the performance over the relay filter optimization only.

C. Rate Maximization

The third design criterion that we consider in this paper is rate maximization. This problem is particularly interesting when high data rates are the main goal of the system design. Again, for this rate maximization problem, we consider joint optimization of the relay filter and the source power allocation. Based on the expressions for the subcarrier SNR and the relay power obtained earlier, the problem is formulated as follows.

Problem 3: For given $\mathbf{f}$, (L_g, σ_g^2), L_r, $P_{s,\max}$, and $P_{r,\max}$

$$\max_{\mathbf{r},P_{s,0},\cdots,P_{s,N-1}} \sum_{k=0}^{N-1} \log\left(1 + \frac{NP_{s,k}\sigma_g^2 \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H \mathbf{r}}{\sigma_r^2 \mathbf{r}^H \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H \mathbf{r} + \sigma_d^2}\right)$$
$$\text{s.t.} \quad \sum_{k=0}^{N-1} P_{s,k} \le P_{s,\max}, \ \mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r} \le P_{r,\max}. \tag{38}$$

When the relay filter is given, an optimal solution to the problem is simply given by the well-known water-filling strategy for parallel Gaussian channels [29]. However, the freedom to design the relay filter and the dependence of the relay transmit power on the source power allocation make the problem far more difficult than a simple water-filling problem. Note that, with $(\mathbf{r}, P_{s,0}, \ldots, P_{s,N-1})$ as the design variable, the problem is a nonconvex problem. Due to the structure of the cost function, it is not easy to convert the problem to a certain convex problem as done earlier. Thus, as in [11], we adopt a direct numerical method to solve this problem based on the PGM, which consists of a gradient descent step for cost reduction and a projection onto the constraint set at each iteration and is widely used for constrained optimization [11], [17]–[19]. To apply the PGM, we rewrite Problem 3 as follows:

$$\min_{\mathbf{r},P_{s,0},\ldots,P_{s,N-1}} -\sum_{k=0}^{N-1} \log\left(1 + \frac{P_{s,k}\mathbf{r}^H \mathbf{Q}_1(k)\mathbf{r}}{\mathbf{r}^H \mathbf{Q}_2(k)\mathbf{r} + \sigma_d^2}\right) \tag{39}$$

$$\text{s.t.} \quad \mathbf{1}^T \mathbf{p} \le P_{s,\max} \tag{40}$$

$$\mathbf{r}^H \mathbf{E}_1 \tilde{\mathbf{\Pi}}^* \mathbf{E}_1^H \mathbf{r} \le P_{r,\max}. \tag{41}$$

where $\mathbf{Q}_1(k) = N\sigma_g^2 \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^T$, $\mathbf{Q}_2(k) = \sigma_r^2 \mathbf{E}_2 \tilde{\mathbf{M}}_k \mathbf{E}_2^H$, $\mathbf{p} = [P_{s,0}, \ldots, P_{s,N-1}]^T$, and $\mathbf{1} = [1, \ldots, 1]^T$. Then, the joint design variable vector $\mathbf{u}$ and the cost function for the PGM are, respectively, given by

$$\mathbf{u} := [\mathbf{p}^T, \mathbf{r}^T]^T \tag{42}$$

$$\phi(\mathbf{u}) := -\sum_{k=0}^{N-1} \log\left(1 + \frac{P_{s,k}\mathbf{r}^H \mathbf{Q}_1(k)\mathbf{r}}{\mathbf{r}^H \mathbf{Q}_2(k)\mathbf{r} + \sigma_d^2}\right). \tag{43}$$

The gradient of $\phi(\mathbf{u})$ with respect to $\mathbf{u}$ can be obtained as

$$\phi'(\mathbf{u}) = -\frac{1}{\ln 2}\left(\sum_{k=0}^{N-1} \frac{1}{1 + P_{s,k}\frac{\mathcal{B}_1(k)}{\mathcal{B}_2(k)}} \cdot \frac{1}{\mathcal{B}_2(k)^2} \times \begin{bmatrix} \mathbf{e}_{k+1}\mathcal{B}_1(k)\mathcal{B}_2(k) \\ P_{s,k}\left(\mathcal{B}_2(k) - \mathcal{B}_1(k)\right)\mathcal{B}_3(k) \end{bmatrix}\right) \tag{44}$$

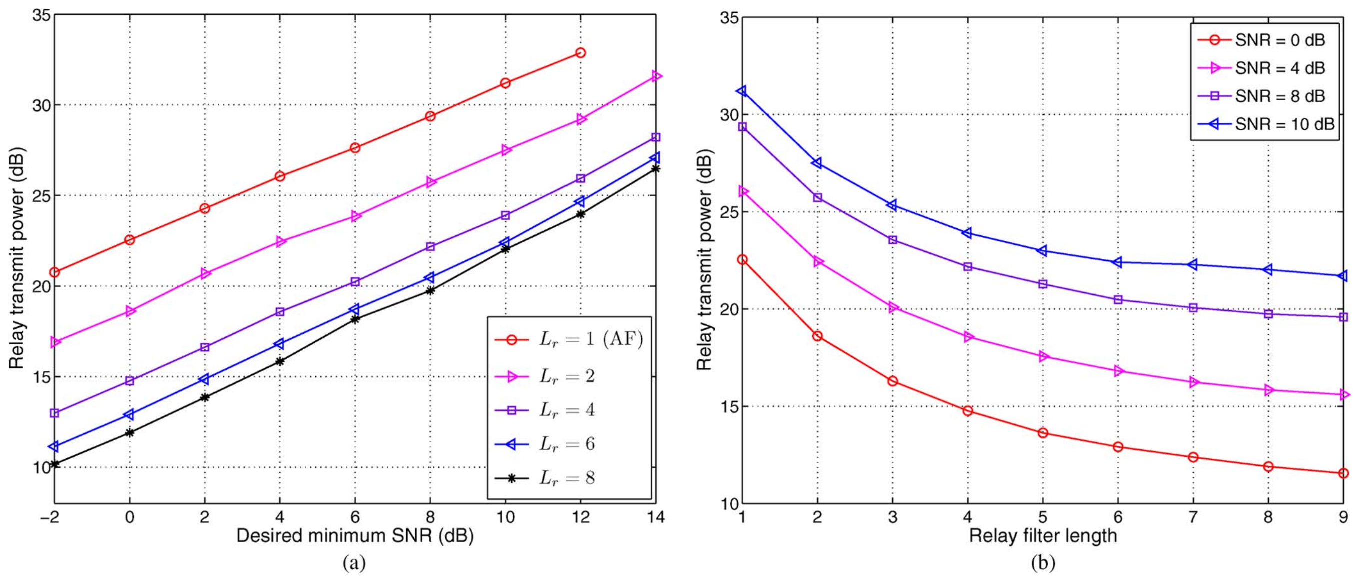


Fig. 3. FF relay transmit power minimization ($P_{s,\max} = 20$ dB, $P_{s,k} = P_{s,\max}/N$, $L_f = L_g = 3$). (a) Relay transmit power versus the desired minimum SNR. (b) Relay transmit power versus the FF relay filter length L_r ($L_r = 1$: AF).

where $\mathcal{B}_1(k) = \mathbf{r}^H \mathbf{Q}_1(k)\mathbf{r}$, $\mathcal{B}_2(k) = \mathbf{r}^H \mathbf{Q}_2(k)\mathbf{r} + \sigma_d^2$, and $\mathcal{B}_3(k) = (\mathbf{Q}_1(k) + \mathbf{Q}_1(k)^T)\mathbf{r}$. The constraint set $\mathcal{S}_1$ defined by (40) is a half-space $\mathcal{H}_{\mathbf{p}}$ for $\mathbf{p}$ defined by a hyperplane with no restriction on $\mathbf{r}$; thus, $\mathcal{S}_1$ is a convex set of $\mathbf{u}$. However, the constraint set S_2 defined by (41) is not convex but biconvex with respect to $\mathbf{p}$ and $\mathbf{r}$. That is, $\mathcal{S}_2$ is an ellipsoid $\xi_{\mathbf{r}}(\mathbf{p})$ for $\mathbf{r}$ for given $\mathbf{p}$, as shown in (41), and is a half-space for $\mathbf{p}$ for given $\mathbf{r}$, as shown in (37). Thus, projection onto $\mathcal{K} := \mathcal{S}_1 \cap \mathcal{S}_2$ can be effectively implemented by successive projections: one projecting $\mathbf{p}$ onto $\mathcal{H}_{\mathbf{p}}$ and the other projecting $\mathbf{r}$ onto the ellipsoid of $\mathbf{r}$ for the given projected $\mathbf{p}$ by the first projection. Based on these projections and (44), we can apply the PGM to Problem 3 in a similar way to that in [11]. It is guaranteed that the PGM yields a unique globally optimal solution when it is applied to a convex optimization problem [19]. However, Problem 3 is not a convex problem; thus, the proposed algorithm does not guarantee convergence to a globally optimal point. However, numerical results show that the algorithm converges and works well.

IV. Numerical Results

Here, we provide some numerical results to evaluate the performance of the FF relay design methods proposed in Section III. We considered a relay network with an OFDM transmitter, an FF relay, and a destination node, as described in Section II. Throughout the simulation, we fixed the number of OFDM subcarriers as $N = 32$ with a minimal cyclic prefix covering the overall FIR channel length in each simulation case. In all cases, both SR and RD channel tap coefficients f_l's and g_l's were generated i.i.d. according to the Rayleigh distribution, i.e., $f_l \overset{\text{i.i.d.}}{\sim} \mathcal{CN}(0,1)$ for $l = 0, 1, \ldots, L_f - 1$ and $g_l \overset{\text{i.i.d.}}{\sim} \mathcal{CN}(0,1)$ for $l = 0, 1, \ldots, L_g - 1$. The relay and the destination had the same noise power $\sigma_r^2 = \sigma_d^2 = 1$, and the source transmit power was 20 dB higher than the noise power, i.e., $P_{s,\max} = 100$. (From here on, all power values in decibels are relative to $\sigma_r^2 = \sigma_d^2 = 1$.)

We first examined the performance of the first FF relay design method, which is provided in Problem 1$'$, to minimize the relay transmit power subject to required SNR constraints on subcarrier channels. Fig. 3 shows the corresponding result. Here, the SR channel length and the RD channel length were set as $L_f = L_g = 3$. We chose $\mathcal{I} = \{0, 1, \ldots, 27\}$ from 32 subcarriers. It is known that, for a set of randomly realized propagation channels, it is not easy to always guarantee the desired SNR for every subcarrier channel when the desired SNR value is high [8]. Thus, in Fig. 3(a) and (b), each line was plotted when Problem 1$'$ was feasible for more than 50% out of 1000 random channel realizations for the given minimum required SNR value for all the subcarrier channels in $\mathcal{I}$, and the plotted value is the relay transmit power averaged over the feasible channel realizations. It is shown that the required relay transmit power for the same minimum SNR required by the FF relay is significantly reduced when compared with that required by the AF relay. Fig. 3(b) shows the relay transmit power versus the relay filter length L_r for various desired minimum SNR values. It is seen that the required relay transmit power for the same desired minimum SNR monotonically decreases with respect to L_r, as expected, and the FF relay achieves most of the gain with only a few FF filter taps.

Next, we evaluated the performance of the second FF relay design method to maximize the worst subcarrier SNR subject to transmit power constraints. First, we considered the relay filter optimization only for a given equal source power allocation, i.e., $P_{s,k} = P_{s,\max}/N$, based on Algorithm 1. Fig. 4 shows the result. For the figure, 500 channels were randomly realized with $L_f = L_g = 3$, and each plotted value is the average over the 500 channel realizations. Here, an OFDM-processing per-subcarrier AF relay is used as an upper

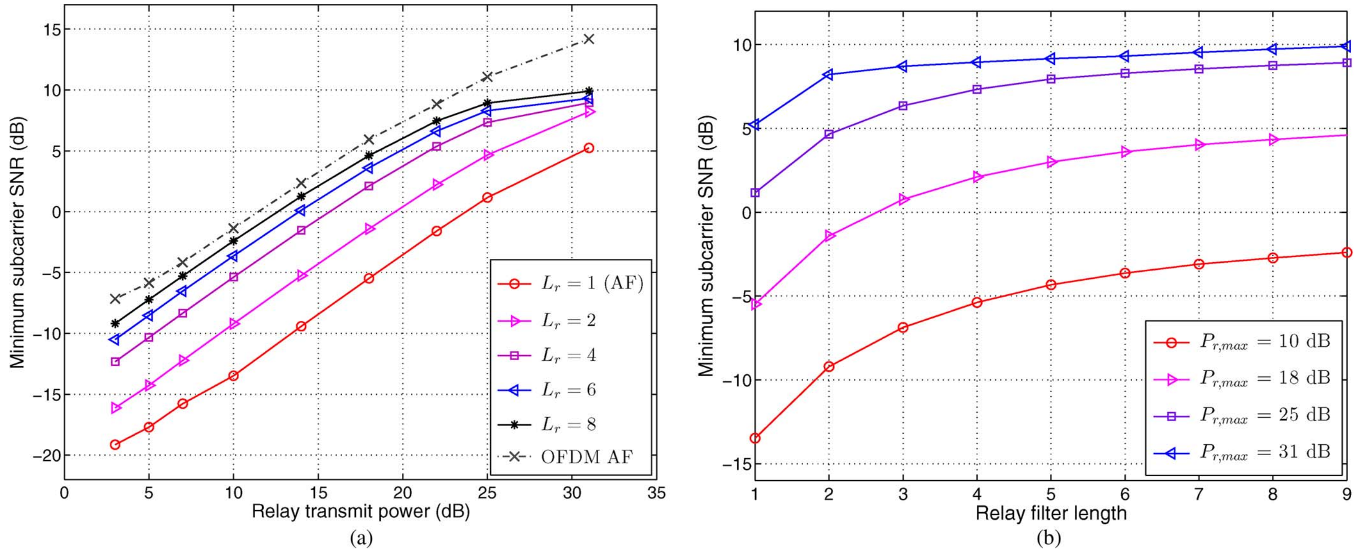


Fig. 4. Worst subcarrier SNR maximization based on Algorithm 1 (relay filter optimization only) ($P_{s,\max} = 20$ dB, $P_{s,k} = P_{s,\max}/N, L_f = L_g = 3$). (a) Worst subcarrier SNR versus the relay transmit power $P_{r,\max}$. (b) Worst subcarrier SNR versus the FF relay filter length L_r (AF: $L_r = 1$).

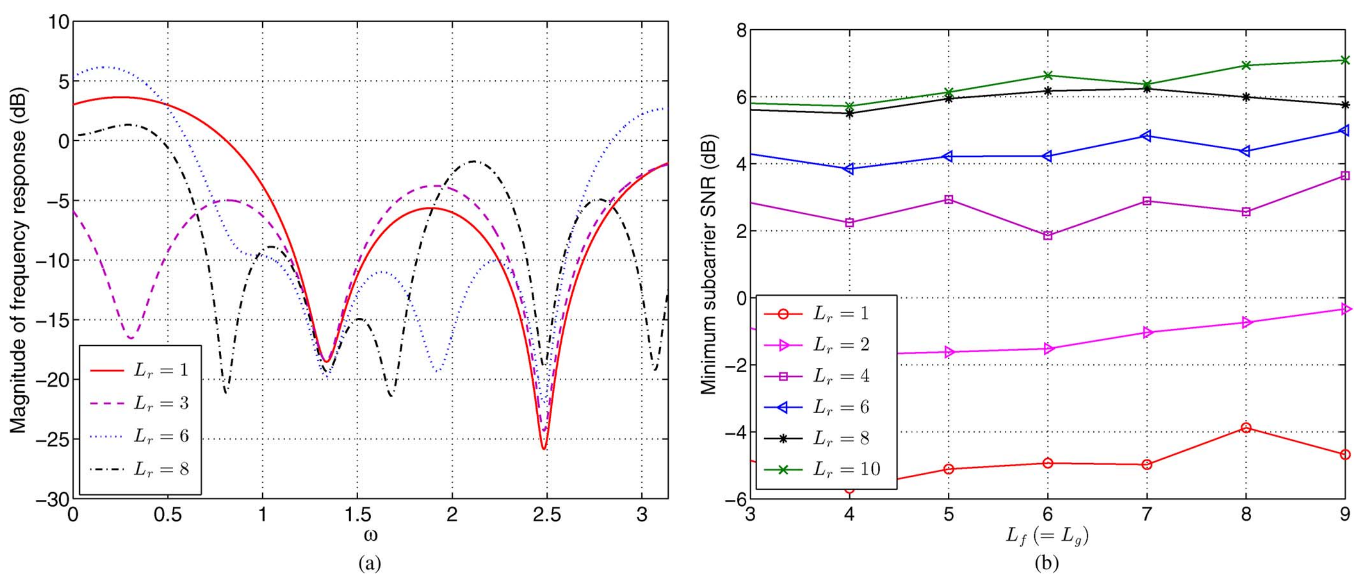
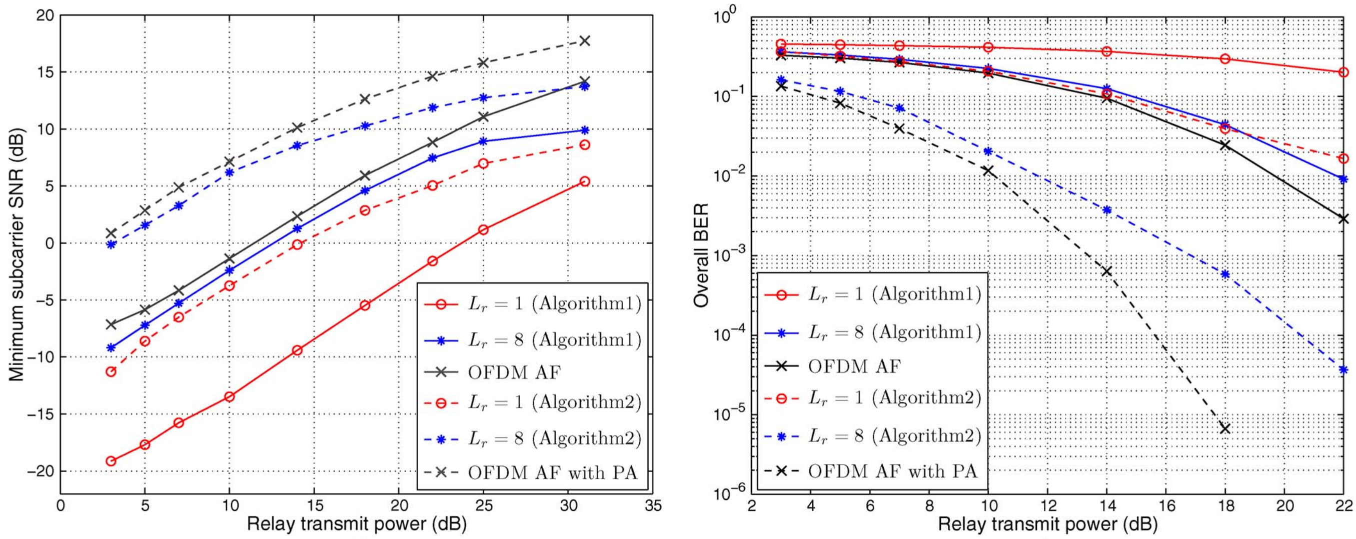


Fig. 5. $P_{s,\max} = 20$ dB, $P_{r,\max} = 20$ dB, $P_{s,k} = P_{s,\max}/N$, and relay filter optimization only by Algorithm 1. (a) Frequency response. (b) Impact of the channel order $L_f = L_g$.

bound of the FF relay.[5] As in the previous case of relay transmit power minimization, the gain by the FF relay over the AF relay $(L_r = 1)$ is significant. Note that, in Fig. 4(a), the performance of the FF relay improves as the FF relay filter length increases and eventually converges to the performance of the OFDM-processing per-subcarrier AF relay designed for the same objective in the range of low and intermediate relay power. This is because what the FF relay does is essentially spectral shaping of the overall channel [see Fig. 5(a)], and this spectral shaping can be maximally done with the OFDM-processing per-subcarrier AF relay. Note that most of the gain is achieved by only a few filter taps for the FF relay, and the performance of the FF relay quickly approaches the upper bound in the range of low and intermediate relay power. On the other hand, it is shown in Fig. 4(a) that the performance of the FF relay saturates in the high relay transmit power range for $L_r = 4$, 6, 8. At high SNR, precise filtering is required to put the relay power exactly on channel notches to maximize the worst subcarrier SNR, and the situation is much stricter than the low SNR case in which the channel notches are immersed in the noise floor. The observed increased gap between the OFDM-processing relay and the FF relay in the high relay power range implies that the proposed algorithm can be easily stuck at some local optimum not at the exact filtering point in the high relay power range. However, the FF scheme still provides far better performance than the AF scheme even in

[5]The derivation of the OFDM-processing per-subcarrier AF relay design for the worst subcarrier SNR maximization is available at http://wisrl.kaist.ac.kr/papers/wisrltechrep2013feb01.pdf.



(a) (b)

Fig. 6. $P_{s,\max} = 20$ dB, $L_f = L_g = 3$. (a) Worst subcarrier SNR versus the relay transmit power $P_{r,\max}$. (b) Overall BER versus $P_{r,\max}$.

this case; furthermore, the practical operating SNR may not be so high to experience this saturation problem.[6] Fig. 5(a) shows several frequency responses of interest for a set of randomly realized channel vectors $\mathbf{f}$ and $\mathbf{g}$ with $L_f = L_g = 3$. ($\mathbf{f} = [-0.0477 + 0.7546i, 0.1938 + 0.2019i, -0.4832 - 0.2111i]$, and $\mathbf{g} = [-0.8370 - 0.2463i, -0.3438 + 0.1734i, -0.5136 + 0.4147i]$.) The frequency response of $f[l] * g[l]$ is the frequency response of the original channel from the source to the destination, and the frequency response of $f[l] * r[l] * g[l]$ is the shaped channel response by the relay filter $r[l]$ designed by Algorithm 1. Note that the notches of the original channel response are filled by the frequency shaping of the relay filter to maximize the worst subcarrier SNR, and this shaping elaborates as the filter order increases. Fig. 5(b) shows the impact of the channel order $L_f = L_g$. In Fig. 5(b), it is deduced that the overall channel order of 5 with $L_f = L_g = 3$ already presents quite complicated frequency selectivity; therefore, the complexity of frequency selectivity caused by higher channel orders does not impact much on the worst subcarrier SNR maximization when the minimum required SNR is not too high.

We next evaluated the performance of the joint source power allocation and FF relay filter design method to maximize the worst subcarrier SNR, which is provided in Algorithm 2, and the result is shown in Fig. 6. Again, the OFDM-processing per-subcarrier AF relay was used as a performance upper bound. It is shown that the joint optimization method significantly outperforms the optimization of the relay filter only presented in Algorithm 1. As aforementioned, the worst subcarrier SNR maximization is closely related to BER minimization. We investigate the BER performance corresponding to Fig. 6(a), and the result is shown in Fig. 6(b). Here, we assumed uncoded quadrature phase-shift keying (QPSK) modulation for each subcarrier channel. From the result in Fig. 6(a), we knew the SNR of each subcarrier channel of the total $N = 32$ subcarrier channels for the designed FF relay filter and source power allocation. Based on this, we computed the subcarrier BER based on the SNR of each subcarrier and averaged all the subcarrier channel BERs to obtain the overall BER. Although there is a noticeable degradation from the OFDM-processing per-subcarrier AF relay at the high SNR, the FF relay significantly improves the BER performance over the AF relay.

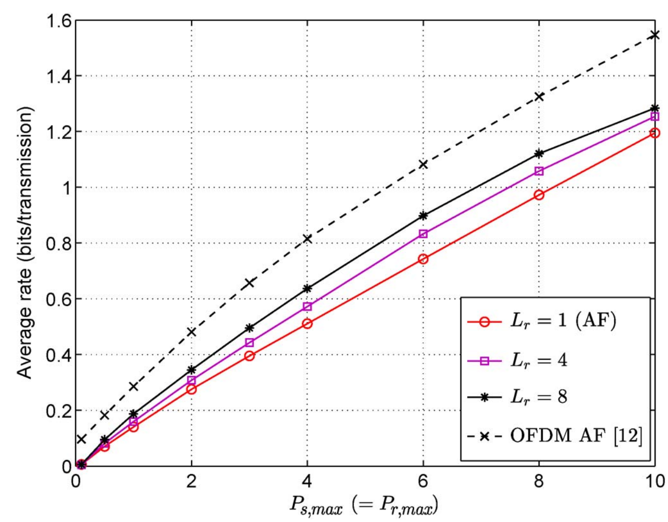


Fig. 7. Sum rate (averaged over 200 channel realizations) versus $P_{s,\max} = P_{r,\max}$. $L_f = L_g = 3$.

Then, we evaluated the performance of the third FF relay design method, which is provided in Section III-C, to maximize the sum rate of subcarrier channels subject to source and relay transmit power constraints. The result is shown in Fig. 7. As a performance upper bound, we considered the method in [12], which does full OFDM processing and subcarrier reordering for rate maximization with full knowledge of the SR and RD channel states. It is observed that the proposed FF relay yields a considerable gain over the AF relay. However, the FF relay

[6]Recall that, with binary PSK or QPSK, the required SNR values for uncoded BER 10^{-3} and 10^{-6} are 6.8 dB and 10.5 dB, respectively. Thus, the saturation around 10-dB minimum SNR may not cause a problem in practical situations.

shows a performance loss when compared with the OFDM processing method. This loss results from the incapability of the FF relay of subcarrier reordering and the lack of the knowledge of the RD channel state.

Finally, we examined the robustness of the proposed FF relay design methods, Algorithm 2, and the proposed rate maximization algorithm against channel information mismatch. We considered two types of channel information error again with $L_f = 3$. One is the RD channel statistic mismatch, and the other is the SR channel state mismatch. For the RD channel statistic mismatch, still, the i.i.d. RD channel model with $L_g = 3$ and $\sigma_g^2 = 1$ for all taps was used to run Algorithm 2 and the proposed rate maximization algorithm, but the true RD channel was generated randomly according to a different channel statistic, i.e., different L_g and/or different channel power profiles. For the SR channel state mismatch, we modeled the available information $\hat{\mathbf{f}}$ for the SR channel $\mathbf{f}$ as $\hat{\mathbf{f}} = \mathbf{f} + \Delta\mathbf{f}$, where $\Delta\mathbf{f} = [\Delta f_0, \Delta f_1, \ldots, \Delta f_{L_f-1}]^T$ is the channel information error vector. Here, the true channel coefficient f_i was generated i.i.d. according to $\mathcal{CN}(0,1)$ for $l = 0, 1, \ldots, L_f - 1$, as aforementioned, and the channel information error Δf_i was generated i.i.d. according to $\Delta f_l \overset{i.i.d.}{\sim} \mathcal{CN}(0, \rho)$ for $l = 0, 1, \ldots, L_f - 1$. Thus, ρ is the relative power of the channel information error to the power of the true channel tap. Fig. 8(a) shows the worst subcarrier SNR obtained by Algorithm 2 (averaged 500 over channel realizations) versus the relay transmit power in the case of RD channel static information mismatch and correct SR channel state information (CSI). It is seen that the proposed FF relay design method is robust against the channel static mismatch. Fig. 8(b) shows the impact of the SR channel mismatch on Algorithm 2 with the correct RD static information. As the number of the FF filter taps increases, there is noticeable performance degradation in the case of the SR channel static mismatch. Fig. 8(c) and (d) shows the impact of the RD channel static mismatch and the SR channel state mismatch on the proposed rate maximization algorithm, respectively. Similar behavior is seen as in the worst subcarrier SNR maximization. In both cases, the algorithms are more robust against the RD static mismatch than against the SR state mismatch. Thus, accurate channel estimation of the SR state is necessary. Fortunately, in most cellular communication systems, there exist pilot signals from the base station that can be used for channel estimation, and the SR channel state can be accurately estimated at the relay by using the pilot signals.

V. Discussion

Here, we discuss several practical issues to implement the proposed full-duplex FF relay. First, let us consider the full-duplex operation. The main advantage of AF relays (i.e., simple repeaters) is that they can be operated in the full-duplex mode, and this full-duplex operation incurs no rate reduction inherent to half-duplexing. In the full-duplex operation, however, we have the problem of self-interference, i.e., the transmitted signal from the relay is fed back to the receiver of the relay. However, this self-interference problem already exists with full-duplex AF relays. There exists vast literature on mitigation of self-interference for AF relays [30]–[33]. It is shown in [30] that echo cancelation combined with the physical separation of transmission and reception antennas at the relay can effectively solve the self-interference problem of full-duplex relays. In the case that interference cancellation is employed at AF relays already, the additional processing for full-duplex FF over full-duplex AF is insignificant because of the already existing up- and downconversion and baseband processing for echo cancelation for full-duplex AF relays.

Next, consider the availability of the channel information assumed earlier. In many research works for relays, it is assumed that all channel information is available at the transmitter and the relay. For a nontransparent relay, this assumption is valid since the relay has its own identity and can transmit its own pilot signal to terminal stations, and the relay can get feedback from terminal stations. However, for the cheap transparent operation, the relay does not have a physical identity and does not receive feedback from terminal stations. Although the transparent relay is invisible to terminal stations, there still exists a control communication link between the base station and the transparent relay in real-world systems for maintenance purposes; Basic relay operation commands from the base station should be delivered to the relay, and operation condition information should be fed back to the base station from the relay. In addition to this base station–relay control communication link, there exists a low-rate robust control link from terminal stations to the base station in all cellular networks.[7] Typically, through this link, channel quality indication and/or CSI is fed back to the base station. Exploiting these two control links and the (typically existing) pilot signal from the base station, one can estimate the necessary channel information assumed earlier, as follows.

Step 1. From the pilot signal $x_p[n]$ transmitted from the base station, the relay estimates the SR channel state $f[l]$ immediately. For example, one can use preamble signals attached in the time domain to OFDM signals. There are several time-domain channel estimation techniques for OFDM signals not requiring ODFM processing.

Step 2. The relay filters the incoming signal with the FIR response $r[l]$ and transmits the filtered signal to the destination.

Step 3. The destination does not know the existence of the relay in the transparent mode, but what the destination receives for the pilot portion is $y_d[n] = x_p[n] * f[n] * r[n] * g[n]$ under the assumption that the SD channel strength is negligible. As usual, the destination node estimates the channel $h[l]$ based on pilot signal $x_p[n]$. The estimated channel at the relay is then $h[l] = f[l] * r[l] * g[l]$.

Step 4. As in most cellular systems, the destination node feeds back the CSI $h[l]$ to the base station via the available uplink control channel.

[7]The control channel typically operates at a low rate. It compensates for low signal power with long bit duration. Thus, although the direct link from the base station to the terminal station is seriously faded, we can still assume that the control link operates properly.

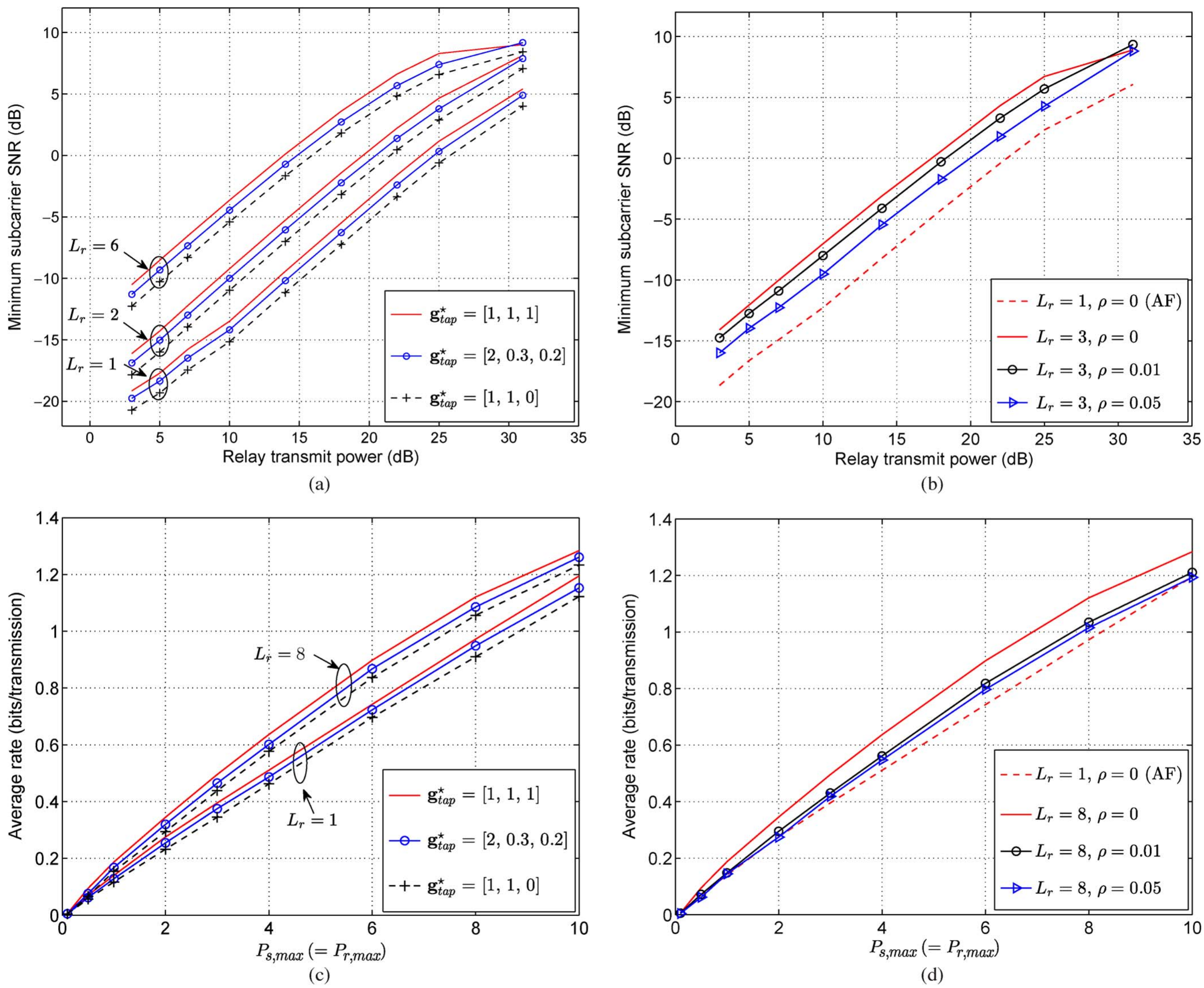


Fig. 8. $P_{s,\max} = P_{r,\max} = 20$ dB, $L_f = L_g = 3$. (a) Worst subcarrier SNR versus $P_{r,\max}$ (Algorithm 2, the RD channel statistic mismatch only). (b) Worst subcarrier SNR versus $P_{r,\max}$ (Algorithm 2, the SR state mismatch only). (c) Rate versus $P_{s,\max} = P_{r,\max}$ (The proposed rate maximization algorithm–the RD channel statistic mismatch only). (d) Rate versus $P_{s,\max} = P_{r,\max}$ (Proposed rate maximization algorithm, the SR channel state mismatch only).

Step 5. The FF relay also feeds back the SR CSI $f[l]$ and its filter response $r[l]$ to the base station via the available control channel between the base station and the relay.

Step 6. The base station now has $f[l]$, $r[l]$, and $h[l] = f[l] * r[l] * g[l]$. It can simply deconvolve $f[l] * r[l]$ from $h[l]$ to obtain $g[l]$. In this stage, it seems more practical and robust to extract and use the stastic of $g[l]$ as in this paper since the CSI is prone to phase errors in the RF circuitries at the relay and communication delays. For example, the delay spread and the channel gain magnitude information can be extracted as aforementioned.

Step 7. The base station computes the FF relay filter response $r[l]$ (based on the results earlier) and downloads the information to the relay via the control channel between the base station and the relay. In this case, the information $r[l]$ does not need to be fed back to the base station from the relay since the base station already has $r[l]$. In this way, computational burden is moved to the base station, and this strategy seems reasonable for the joint optimization considered in Section III-B and C.

As shown earlier, a practical implementation of the proposed FF scheme is possible and does not require any standard change.

Finally, we consider the possibility of extension of the FF relay to the broadcasting situation in which the base station serves several terminal stations in the relay cell simultaneously. In this paper, we assumed CSI for the SR channel and channel statistic information for the RD channel. The assumption of channel statistic information for the RD channel makes the proposed FF relay design method useful for broadcasting. Suppose that the base station acquires the channel information from each terminal station by the given method. Then, the base station has the RD channel information from all terminal stations in the relay cell. The base station can select and schedule terminal stations with similar channel statistics and designs the relay filter accordingly. In this way, the proposed FF relay scheme can be applied to the broadcasting scenario since the proposed design methods do not require exact CSI or the phase of the channel gain for the RD channel and are robust against the RD channel static mismatch. In this broadcasting scenario, the worst subcarrier SNR maximization in Section III-B improves QoS fairness among users, and the rate maximization in Section III-C increases the overall system sum rate.

VI. Conclusion

In this paper, we have considered the FF relay design for OFDM systems for transparent relay operation to compromise the performance and complexity between the simple repeater and the full OFDM-processing relay. We have considered three FF relay design criteria of minimizing the relay transmit power subject to per-subcarrier SNR constraints, maximizing the worst subcarrier SNR subject to transmit power constraints, and maximizing the data rate subject to transmit power constraints. We have proposed an efficient algorithm for each of the three criteria based on convex relaxation, alternating optimization, and the PGM. The proposed FF relay significantly outperforms the simple repeater with a slight increase in complexity and the same operating condition and thus provides an effective alternative to the simple repeater. In this paper, we assumed SISO-OFDM systems. However, most current OFDM systems employ MIMO communications; thus, extension to the MIMO case is left as future work.

Appendix
Proof of Theorem 1

By introducing slack variable τ, we convert Problem $1'$ to the following equivalent problem:

$$\min_{\tau,\mathcal{R}} \quad \tau \tag{45}$$

$$\text{s.t.} \quad \mathrm{tr}(\mathbf{\Phi}_P\mathcal{R}) \le \tau \tag{46}$$

$$\mathrm{tr}\left([\mathbf{\Phi}_S(k) - \gamma_k\mathbf{\Phi}_N(k)]\,\mathcal{R}\right) \ge \sigma_d^2\gamma_k \tag{47}$$

$$\tau \ge 0,\ k = 0,\ldots,N-1 \tag{48}$$

$$\mathcal{R} \succeq 0. \tag{49}$$

The Lagrange dual function for the given problem is given by

$$g(\lambda,\{\mu_k\},\nu,\mathbf{\Psi}) = \inf_{\tau,\mathcal{R}}\left((1-\lambda-\nu)\tau + \sum_{k=0}^{N-1}\mu_k\gamma_k\sigma_d^2 + \mathrm{tr}\left(\left\{\underbrace{\lambda\mathbf{\Phi}_P - \sum_{k=0}^{N-1}\mu_k\left[\mathbf{\Phi}_S(k) - \gamma_k\mathbf{\Phi}_N(k)\right] - \mathbf{\Psi}}_{=:\mathbf{Q}(\lambda,\{\mu_k\})}\right\}\mathcal{R}\right)\right) \tag{50}$$

where $\lambda \ge 0$, $\{\mu_k \ge 0\}$, $\nu \ge 0$, and $\mathbf{\Psi} \succeq 0$ are the dual variables associated with (46)–(49), respectively. If $1-\lambda-\nu \neq 0$ or $\mathbf{Q}(\lambda,\{\mu_k\}) - \mathbf{\Psi} \neq 0$, then the dual function value is minus infinity, or we have trivial solutions $\tau = 0$ and/or $\mathcal{R} = \mathbf{0}$. Thus, for the nontrivial feasibility of τ and $\mathcal{R}$, we have $1-\lambda = \nu(\ge 0)$ and $\mathbf{Q}(\lambda,\{\mu_k\}) = \mathbf{\Psi}(\succeq 0)$. Then, the Lagrange dual function is easily obtained as $g(\lambda,\{\mu_k\},\nu,\mathbf{\Psi}) = \sum_{k=0}^{N-1}\mu_k\gamma_k\sigma_d^2$, and the corresponding dual problem is given by

$$\begin{aligned}\max_{\lambda,\{\mu_k\}} \quad & \sum_{k=0}^{N-1}\mu_k\gamma_k\sigma_d^2 \\ \text{s.t.} \quad & 0 \le \lambda \le 1,\ \mathbf{Q}(\lambda,\{\mu_k\}) \succeq 0, \\ & \mu_k \ge 0,\ k = 0,1,\ldots,N-1.\end{aligned} \tag{51}$$

Let $\lambda^\star$, $\{\mu_k^\star\}$, $\tau^\star$, $\mathbf{Q}^\star$, and $\mathcal{R}^\star$ be the optimal values for the problem. ($\nu^\star$ and $\mathbf{\Psi}^\star$ are automatically determined based on these quantities. The dependence of $\mathbf{Q}$ on λ and $\{\mu_k\}$ is not explicitly shown for notational simplicity from here on.) From the complementary slackness conditions for (46) and (47), we have

$$\lambda^\star\left(\mathrm{tr}\left(\mathbf{\Phi}_P\mathcal{R}^\star\right) - \tau^\star\right) + \sum_{k=0}^{N-1}\mu_k^\star\left(\gamma_k\sigma_d^2 - \mathrm{tr}\left(\left[\mathbf{\Phi}_S(n) - \gamma_k\mathbf{\Phi}_N(k)\right]\mathcal{R}^*\right)\right) = 0 \tag{52}$$

which is equivalent to

$$\sum_{k=0}^{N-1}\mu_k^\star\gamma_k\sigma_d^2 - \lambda^\star\tau^\star + \mathrm{tr}\left(\left\{\underbrace{\lambda^\star\mathbf{\Phi}_P - \sum_{k=0}^{N-1}\mu_k^\star\left[\mathbf{\Phi}_S(k) - \gamma_k\mathbf{\Phi}_N(k)\right]}_{=\mathbf{Q}^\star}\right\}\mathcal{R}^\star\right) = 0. \tag{53}$$

Since the problem (45)–(49) is a convex optimization problem, the duality gap is zero, i.e., $\sum_{k=0}^{N-1}\mu_k^\star\gamma_k\sigma_d^2 = \tau^\star$. Thus, both the first and second terms in the left-hand side of (53) are nonnegative since $\sum_{k=0}^{N-1}\mu_k^\star\gamma_k\sigma_d^2 - \lambda^\star\tau^\star = \tau^\star(1-\lambda^\star) \ge 0$, and $\mathrm{tr}(\mathbf{Q}^\star\mathcal{R}^\star) \ge 0$. (The trace of the product of two positive semidefinite matrices is nonnegative [34].) Therefore, $\lambda^\star = 1$ and $\mathrm{tr}(\mathbf{Q}^\star\mathcal{R}^\star) = 0$. It is obvious that $\mathbf{Q}^\star \not\succ 0$ for a nontrivial $\mathcal{R}^\star$ from $\mathrm{tr}(\mathbf{Q}^\star\mathcal{R}^\star) = 0$, i.e., the $L_r \times L_r$ matrix $\mathbf{Q}^\star$ does not have full rank. This is because $\mathrm{tr}(\mathbf{Q}^\star\mathcal{R}^\star) = \sum_i \sigma_i\mathrm{tr}(\mathbf{Q}^\star\mathbf{u}_i\mathbf{u}_i^H) = \sum_i \sigma_i(\mathbf{u}_i^H\mathbf{Q}^\star\mathbf{u}_i)$, where $\mathcal{R} = \sum_i \sigma_i\mathbf{u}_i\mathbf{u}_i^H$ is the eigendecomposition of $\mathcal{R}^\star$. (If $\mathbf{Q}^\star \succ 0$, then $\mathrm{tr}(\mathbf{Q}^\star\mathcal{R}^\star) > 0$.) Note from (53) that

$$\mathbf{Q}^\star = \mathbf{\Phi}_P - \sum_{k=0}^{N-1}\mu_k^\star\left(\mathbf{\Phi}_S(k) - \gamma_k\mathbf{\Phi}_N(k)\right) \tag{54}$$

where $\mathbf{\Phi}_P$ is a positive definite matrix defined in (24), $\mathbf{\Phi}_N(k)$ is a positive semi-definite matrix defined in (21), and $\mathbf{\Phi}_S(k)$, defined in (19), is a rank-one matrix by Lemma 2. Now, under the assumption that all the SNR constraints except one are satisfied with strict inequality, we have $\mu_i \neq 0$ for some i and $\mu_j = 0\ \forall j \neq i$ from the complementary slackness conditions. In this case, $\mathbf{Q}^\star$ is given by

$$\mathbf{Q}^\star = \underbrace{\boldsymbol{\Phi}_P + \mu_i^\star \gamma_i \boldsymbol{\Phi}_N(i)}_{\text{rank } L_r} - \underbrace{\mu_i^\star \boldsymbol{\Phi}_S(i)}_{\text{rank1}}. \quad (55)$$

Due to the structure of $\mathbf{Q}^\star$ in (55), the rank of $\mathbf{Q}^\star$ is larger than or equal to $L_r - 1$. Since $\mathbf{Q}^\star \not\succeq 0$, $\text{rank}(\mathbf{Q}^\star) = L_r - 1$. Since $0 = \text{tr}(\mathbf{Q}^\star \mathcal{R}^\star) = \sum_{i=1}^{L_r-1} \eta_i (\mathbf{v}_i^H \mathcal{R}^\star \mathbf{v}_i)$ (where $\mathbf{Q}^\star = \sum_{i=1}^{L_r-1} \eta_i \mathbf{v}_i \mathbf{v}_i^H$ is the eigendecomposition of $\mathbf{Q}^\star$), we conclude that $\mathcal{R}^\star$ has nullity $L_r - 1$ and thus has rank one under the assumption of Theorem 1. ■

Lemma 2: If $N - L_f + 1 > L_f + L_g - 1$, $\boldsymbol{\Phi}_S(k)$ has rank one regardless of the value of k.

Proof of Lemma 2: Recall that [see (19) and (26)]

$$\left(NP_{s,k}\sigma_g^2\right)^{-1} \boldsymbol{\Phi}_S(k) = \mathbf{E}_1 \tilde{\mathbf{K}}_k \mathbf{E}_1^H, \tilde{\mathbf{K}}_k = \tilde{\mathbf{I}}_{L_g} \otimes \mathbf{K}_k, \mathbf{K}_k = \mathbf{F}^* \mathbf{T}^* \mathbf{w}_k \mathbf{w}_k^H \mathbf{T}^T \mathbf{F}^T. \quad (56)$$

Let $\mathbf{E}_1$ in (20) be partitioned as

$$\mathbf{E}_1 = \left[\mathbf{E}_1^{(1)}, \mathbf{E}_1^{(2)}, \ldots, \mathbf{E}_1^{(N+L_g-1)}\right].$$

Note that $\mathbf{F}$ and $\mathbf{T}$ are Toeplitz matrices and that $\mathbf{w}_k \mathbf{w}_k^H$ is also a Toeplitz matrix regardless of k due to the property of DFT matrices. It is not difficult to show that $\mathbf{K}_k(1 : N - L_f + 1, 1 : N - L_f + 1)$ is a Toeplitz matrix, where $\mathbf{A}(a : b, c : d)$ denotes a submatrix of $\mathbf{A}$ composed of the rows from a to b and columns from c to d. Now, $\boldsymbol{\Phi}_S(k)$ can be rewritten as

$$\left(NP_{s,k}\sigma_g^2\right)^{-1} \boldsymbol{\Phi}_S(k) = \mathbf{E}_1^{(1)} \mathbf{K}_k \mathbf{E}_1^{(1)H} + \mathbf{E}_1^{(2)} \mathbf{K}_k \mathbf{E}_1^{(2)H} + \cdots + \mathbf{E}_1^{(L_g)} \mathbf{K}_k \mathbf{E}_1^{(L_g)H}. \quad (57)$$

Here, the operation $\mathbf{E}_1^{(i)} \mathbf{K}_k \mathbf{E}_1^{(i)H}$ extracts a $L_r \times L_r$ submatrix $\mathbf{K}_k(i : L_r + i - 1, i : L_r + i - 1)$ from $\mathbf{K}_k$. If $N - L_r + 1 > L_r + L_g - 1$, this operation extracts the same submatrix from $\mathbf{K}_k$ regardless of i since $\mathbf{K}_k(1 : N - L_r + 1, 1 : N - L_r + 1)$ is a Toeplitz matrix. Thus, we have

$$\begin{aligned}\left(NP_{s,k}\sigma_g^2\right)^{-1} \boldsymbol{\Phi}_S(k) &= L_g \mathbf{E}_1^{(1)} \mathbf{K}_k \mathbf{E}_1^{(1)H} \\ &= L_g \left(\mathbf{E}_1^{(1)} \mathbf{F}^* \mathbf{T}^* \mathbf{w}_k\right)\left(\mathbf{E}_1^{(1)} \mathbf{F}^* \mathbf{T}^* \mathbf{w}_k\right)^H\end{aligned} \quad (58)$$

and $\boldsymbol{\Phi}_S(k)$ has rank one if the condition $N - L_f + 1 > L_t + L_g - 1$. ■

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