Hybrid Beamformer Design for mmWave Wideband Multi-User MIMO-OFDM Systems

(Invited Paper)

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***Abstract*—In this paper, analog-and-digital hybrid beamformer design is considered for millimeter-wave (mmWave) wideband multi-user (MU) multiple-input single-output (MISO)-OFDM downlink systems, where a single common analog beamformer is used for all subcarriers while a separate digital beamformer is applied to each subcarrier. By applying a signal-to-leakage-plus-noise ratio (SLNR) approach, an efficient beamformer design algorithm based on a trace quotient is derived for joint design of the common analog beamformer and the separate digital beamformers.**

***Index Terms*—Hybrid beamforming, millimeter-wave, wideband, MIMO-OFDM**

I. Introduction

Hybrid beamforming is a practical method for beamformer implementation for massive multiple-input multiple-output (MIMO) in the mmWave band [1]. In (narrow-band) hybrid beamforming, the overall beamformer is decomposed into a digital beamformer in the digital domain and an analog beamformer in the analog domain, and a limited number of radio-frequency (RF) chains far less than the number of antennas is used with the aid of the analog beamformer without much performance loss compared to fully digital beamforming. Recently, there has been extensive work for hybrid beamformer design from academia and industry [1], [2] mostly for narrow-band hybrid beamformer design [3], [4], [5]. Unlike the narrow-band case in which only a single analog beamformer and a single digital beamformer need to be considered, in wideband hybrid beamforming a single analog beamformer is common to all subcarriers due to the limitation of analog processing and a separate digital beamformer is used for each subcarrier to adapt the beamformer to a different channel at each subcarrier. Thus, the common analog beamformer should be designed with consideration of impact to all subcarriers and the design problem becomes more complicated than the narrow-band case. There have been some works for wideband hybrid beamforming, e.g., [6], [7], [8], but these works are for the single-user case in which there is no inter-user interference.

In this paper, we consider mmWave wideband hybrid beamformer design for MU-MISO-OFDM downlink systems. In the MU case, each subcarrier supports multiple users with digital beamforming, and the combined signal of multiple subcarriers goes through a common analog beamformer. Thus, in the MU hybrid beamforming case, not only the signal power of each user but also interference from other users should be incorporated into beamformer design. However, direct use of signal-to-interference-plus-noise ratio (SINR) as the design criterion is difficult since it yields a non-convex optimization problem [9]. Hence, we here apply the SLNR approach often used for MU-MIMO downlink beamformer design [9], [10], and derive an efficient algorithm for joint design of the common analog beamformer and separate digital beamformers for wideband MU-MISO-OFDM downlink.

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Notations: Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^T$, $\mathbf{A}^H$, and $\text{Tr}(\mathbf{A})$ indicate the transpose, conjugate transpose, and trace of $\mathbf{A}$, respectively. $\text{diag}(\mathbf{A}_1, \cdots, \mathbf{A}_K)$ denotes the diagonal matrix with diagonal elements $\mathbf{A}_1, \cdots, \mathbf{A}_K$. $\|\mathbf{b}\|$ denotes the 2-norm of vector $\mathbf{b}$. $\mathbf{I}$ and $\mathbf{0}$ denote an identity matrix and a matrix with zero elements, respectively. $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is circularly-symmetric complex Gaussian-distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. $\otimes$ denotes the Kronecker product. $\mathbb{C}$ is the set of complex numbers. $\iota := \sqrt{-1}$.

II. System Model

We consider hybrid beamforming for a mmWave MU-MISO-OFDM single-cell downlink system, where a base station (BS) with N_t transmit antennas and N_a RF chains serves K single-antenna users for each subcarrier, and OFDM with N_f subcarriers is applied to transmission and reception. We assume that the number of RF chains is less than the number of transmit antennas, and the number of users for each subcarrier is less than or equal to the number of RF chains i.e., $K \le N_a < N_t$. We assume that the analog beamformer is common to all N_f subcarriers and a separate digital beamformer is used for each subcarrier. From here on, we derive the data model step by step.

Transmitter: First, the overall transmitted symbols for all subcarriers and all users are represented as a single vector given by

$$\mathbf{s} = [\mathbf{s}_1^T, \mathbf{s}_2^T, \cdots, \mathbf{s}_{N_f}^T]^T, \tag{1}$$

where $\mathbf{s}_i = [s_{1,i}, \cdots, s_{K,i}]^T \in \mathbb{C}^{K\times 1}$ is the symbol vector for the i-th subcarrier, $i = 1, \cdots, N_f$, containing K data symbols for the K served users at the i-th subcarrier. We assume that the total transmit power P is equally divided for each symbol as $|s_{k,i}|^2 = P/(KN_f)$. The transmit symbol vector $\mathbf{s}_i$ for the i-th subcarrier is digitally precoded in the digital domain as

$$\mathbf{B}_i\mathbf{s}_i, \quad i = 1, \cdots, N_f,$$

where $\mathbf{B}_i = [\mathbf{b}_{1,i}, \mathbf{b}_{2,i}, \cdots, \mathbf{b}_{K,i}]$ is the $N_a \times K$ digital precoding matrix for the i-th subcarrier. Thus, the overall digitally-precoded symbol vector can be represented as

$$\mathbf{Bs} = \begin{bmatrix} \mathbf{B}_1\mathbf{s}_1 \\ \mathbf{B}_2\mathbf{s}_2 \\ \vdots \\ \mathbf{B}_{N_f}\mathbf{s}_{N_f} \end{bmatrix}, \tag{2}$$

where $\mathbf{B}$ is the overall digital precoding matrix given by

$$\mathbf{B} = \text{diag}(\mathbf{B}_1, \mathbf{B}_2, \ldots, \mathbf{B}_{N_f}). \tag{3}$$

The digitally-precoded signal $\mathbf{Bs}$ in the frequency domain is converted to a time-domain signal by inverse discrete-Fourier transform (IDFT), given by

$$\tilde{\mathbf{s}} = \left(\mathbf{F}^H \otimes \mathbf{I}_{N_a}\right)\mathbf{Bs}, \tag{4}$$

where $\mathbf{F}$ is the DFT matrix of size N_f whose element (i,j) is given by $\frac{1}{\sqrt{N_f}}e^{-\iota 2\pi ij/N_f}$. A cyclic-prefix (CP) of size N_{cp} is added to the time-domain signal $\tilde{\mathbf{s}}$ to generate the CP-added signal $\tilde{\mathbf{s}}_{cp}$. The CP-added digital signal $\tilde{\mathbf{s}}_{cp}$ is converted to the analog domain. Then, the analog-domain converted signal $\tilde{\mathbf{s}}_{cp}$ is analog-precoded and transmitted from the BS. By concatenating the time-domain signal from sample time $-N_{cp}$ to N_f, the BS transmitted signal vector is expressed as

$$\tilde{\mathbf{x}} = \left(\mathbf{I}_{N_f+N_{cp}} \otimes \mathbf{A}\right)\tilde{\mathbf{s}}_{cp}, \tag{5}$$

where $\mathbf{A} = [\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_{N_a}] \in \mathbb{C}^{N_t \times N_a}$ is the common analog beamforming matrix. We assume that each column of $\mathbf{A}$ has unit norm, i.e., $\mathbf{a}_r^H\mathbf{a}_r = 1$ for $r = 1, \cdots, N_a$, and equal power is allocated to each symbol, i.e., $\|\mathbf{Ab}_{k,i}\|^2 = 1$ for all $k = 1, \cdots, K$ and $i = 1, \cdots, N_f$, where $\mathbf{b}_{k,i}$ is the k-th column of the digital beamforming matrix $\mathbf{B}_i$ for the i-th subcarrier.

Channel: We assume a wideband MU-MISO channel model. In this model, the channel from the BS to the k-th user served at the i-th subcarrier, denoted by user $k^{(i)}$, is given by a D-tap MISO finite-duration impulse response (FIR) $\{\mathbf{g}_{k^{(i)}}^H(0), \cdots, \mathbf{g}_{k^{(i)}}^H(D-1)\}$, where $D \le N_{cp}$. We further assume that the m-th tap $1 \times N_t$ MISO channel vector $\mathbf{g}_{k^{(i)}}^H(m)$ of user $k^{(i)}$ follows the geometric model with L_s scatterers:

$$\mathbf{g}_{k^{(i)}}^H(m) = \sqrt{\frac{N_t}{L_s}}\sum_{l=1}^{L_s}\alpha_{k^{(i)},m,l}\mathbf{u}_t^H(\phi_{k^{(i)},m,l}), \tag{6}$$

where $\alpha_{k^{(i)},m,l}$ and $\phi_{k^{(i)},m,l}$ is the complex gain and angle-of-departure (AoD) at the m-th delay tap and the l-th path of the channel of user $k^{(i)}$, and $\mathbf{u}_t(\phi_{k^{(i)},m,l})$ is the antenna array response of the transmitter given under uniform linear array (ULA) assumption by $\mathbf{u}_t(\phi_{k^{(i)},m,l}) = (1/\sqrt{N_t}) \cdot \left[1,\ \omega^1(\phi_{k^{(i)},m,l}),\ \cdots,\ \omega^{N_t-1}(\phi_{k^{(i)},m,l})\right]^T$. Here, $\omega^n(\phi_{k^{(i)},m,l}) = e^{\iota n \frac{2\pi}{\lambda} d \sin(\phi_{k^{(i)},m,l})}$, d is the distance between two adjacent antenna elements, and λ is the carrier wavelength. Note that for the wide-band channel model, each delay tap may have different propagation paths. Note also that in the narrow-band channel case, we only have $\mathbf{g}_{k^{(i)}}^H(0)$.

Receiver: The BS transmitted signal $\tilde{\mathbf{x}}$ goes through the channel described above to each user and the received signal at each user is corrupted by additive white Gaussian noise (AWGN) at each user's receiver. Then, the received signal of user $k^{(i)}$ after removal of the CP portion can be written as

$$\tilde{\mathbf{y}}_{k^{(i)}} = \mathbf{G}_{k^{(i)}}\left(\mathbf{I}_{N_f} \otimes \mathbf{A}\right)\left(\mathbf{F}^H \otimes \mathbf{I}_{N_a}\right)\mathbf{Bs} + \tilde{\mathbf{n}}_{k^{(i)}}, \tag{7}$$

where $\mathbf{G}_{k^{(i)}}$ is the $N_f \times N_f N_t$ block circulant channel filtering matrix for user $k^{(i)}$ with $[\mathbf{g}_{k^{(i)}}(0),\ \cdots,\ \mathbf{g}_{k^{(i)}}(D-1),\ \mathbf{0},\ \cdots,\ \mathbf{0}]^H$ as its first block column, and $\tilde{\mathbf{n}}_{k^{(i)}} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$ is AWGN. Applying DFT to $\tilde{\mathbf{y}}_{k^{(i)}}$, we obtain the received signal in the frequency domain as

$$\begin{aligned} \mathbf{y}_{k^{(i)}} &= \mathbf{F}\left[\mathbf{G}_{k^{(i)}}(\mathbf{I}_{N_f} \otimes \mathbf{A})(\mathbf{F}^H \otimes \mathbf{I}_{N_a})\mathbf{Bs} + \tilde{\mathbf{n}}_{k^{(i)}}\right] \\ &= \mathbf{H}_{k^{(i)}}\mathbf{Bs} + \mathbf{n}_{k^{(i)}}, \end{aligned} \tag{8}$$

where $\mathbf{H}_{k^{(i)}} = \mathbf{FG}_{k^{(i)}}(\mathbf{I}_{N_f} \otimes \mathbf{A})(\mathbf{F}^H \otimes \mathbf{I}_{N_a})$ and $\mathbf{n}_{k^{(i)}} = \mathbf{F}\tilde{\mathbf{n}}_{k^{(i)}} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$.

Proposition 1: Based on the properties of circulant matrices, $\mathbf{H}_{k^{(i)}}$ in (8) can be written as

$$\mathbf{H}_{k^{(i)}} = \begin{bmatrix} \mathbf{h}_{k^{(i)},1}^H\mathbf{A} & & 0 \\ & \ddots & \\ 0 & & \mathbf{h}_{k^{(i)},N_f}^H\mathbf{A} \end{bmatrix}, \tag{9}$$

where $\mathbf{h}_{k^{(i)},j}^H$ is the $1 \times N_t$ effective MISO channel vector at the j-th subcarrier of user $k^{(i)}$, given by

$$\mathbf{h}_{k^{(i)},j}^H = [\lambda_{k^{(i)},j}^{(1)}, \cdots, \lambda_{k^{(i)},j}^{(N_t)}]. \tag{10}$$

Here, $\lambda_{k^{(i)},j}^{(n)}$ is the j-th eigenvalue of $\mathbf{G}_{k^{(i)}}^{(n)}$ which is the submatrix of $\mathbf{G}_{k^{(i)}}$ composed of the columns $n, n+N_t, n+2N_t, \cdots, n+(N_f-1)N_t$ of $\mathbf{G}_{k^{(i)}}$ for $n = 1, \cdots, N_t$. That is, $\mathbf{FG}_{k^{(i)}}^{(n)}\mathbf{F}^H = \mathbf{\Lambda}_{k^{(i)}}^{(n)} = \text{diag}(\lambda_{k^{(i)},1}^{(n)},\ \ldots,\ \lambda_{k^{(i)},N_f}^{(n)})$.

Proof: See Appendix. ∎

Applying (9), we have $\mathbf{y}_{k^{(i)}}$ in (8) rewritten as

$$\mathbf{y}_{k^{(i)}} = \begin{bmatrix} \mathbf{h}_{k^{(i)},1}^H\mathbf{AB}_1\mathbf{s}_1 \\ \mathbf{h}_{k^{(i)},2}^H\mathbf{AB}_2\mathbf{s}_2 \\ \vdots \\ \mathbf{h}_{k^{(i)},N_f}^H\mathbf{AB}_{N_f}\mathbf{s}_{N_f} \end{bmatrix} + \mathbf{n}_{k^{(i)}}. \tag{11}$$

Finally, the received symbol of user $k^{(i)}$ at the i-th subcarrier is given by (recall that users $k^{(i)}$, $k = 1, \cdots, K$ are served at the i-th subcarrier)

$$\begin{aligned} y_{k^{(i)},i} &= \mathbf{h}_{k^{(i)},i}^H\mathbf{AB}_i\mathbf{s}_i + n_{k^{(i)},i} \\ &= \mathbf{h}_{k^{(i)},i}^H\mathbf{Ab}_{k,i}s_{k,i} + \mathbf{h}_{k^{(i)},i}^H\sum_{\substack{p=1 \\ p\neq k}}^{K}\mathbf{Ab}_{p,i}s_{p,i} + n_{k^{(i)},i}, \end{aligned} \tag{12}$$

where $n_{k^{(i)},i}$ is the i-th element of the noise vector $\mathbf{n}_{k^{(i)}}$.

III. Problem Formulation and Beamformer Design

Typically-considered sum-rate-maximizing beamformer design can be formulated under the considered constraints as follows:

$$\{\mathbf{A}^o, \{\mathbf{B}_i^o\}_{i=1}^{N_f}\} = \underset{\mathbf{A}, \{\mathbf{B}_i\}_{i=1}^{N_f}}{\arg\max} \sum_{k=1}^{K}\sum_{i=1}^{N_f} \log(1+\text{SINR}_{k,i}) \quad (13)$$
$$\text{s.t.} \;\; \mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1,\cdots,N_a$$
$$\|\mathbf{A}\mathbf{b}_{k,i}\|^2 = 1,\ \forall k=1,\cdots,K,\ \forall i = 1,\cdots,N_f,$$

where the SINR of user $k^{(i)}$ at the i-th subcarrier is given by

$$\text{SINR}_{k,i} = \frac{\rho|\mathbf{h}_{k^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2}{\rho\sum_{p\neq k}|\mathbf{h}_{k^{(i)},i}^H\mathbf{A}\mathbf{b}_{p,i}|^2+1} \quad (14)$$

with $\rho = P/(N_aN_f\sigma^2)$. However, the problem (13) is not a convex problem and is difficult to solve. To circumvent this difficulty, we apply the SLNR approach often used in MU-MIMO beamformer design [9], [10]. Although several formulations are possible with this approach, we formulate the problem of maximizing the sum of SLNRs of all users and subcarriers:

$$\{\mathbf{A}^\star, \{\mathbf{B}_i^\star\}_{i=1}^{N_f}\} = \underset{\mathbf{A}, \{\mathbf{B}_i\}_{i=1}^{N_f}}{\arg\max} \sum_{k=1}^{K}\sum_{i=1}^{N_f} \text{SLNR}_{k,i} \quad (15)$$
$$\text{s.t.} \;\; \mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1,\cdots,N_a$$
$$\|\mathbf{A}\mathbf{b}_{k,i}\|^2 = 1,\ \forall k=1,\cdots,K,\ \forall i = 1,\cdots,N_f,$$

where the SLNR for user $k^{(i)}$ at the i-th subcarrier is given by

$$\text{SLNR}_{k,i} = \frac{\rho|\mathbf{h}_{k^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2}{\rho\sum_{p\neq k}|\mathbf{h}_{p^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2+1}. \quad (16)$$

Note the difference between (14) and (16). Whereas the first term in the denominator of the right-hand side (RHS) of (14) is the interference from other users to the k-th user, the first term in the denominator of the RHS of (16) is the leakage to other users from the k-th user. The key fact about SLNR is that both the numerator and the denominator have the same term $\mathbf{A}\mathbf{b}_{k,i}$ and this fact facilitates solving the problem. Note that for given $\mathbf{A}$, $\text{SLNR}_{k,i}$ is affected only by $\mathbf{b}_{k,i}$. Hence, our approach to the problem (15) is first to express each $\text{SLNR}_{k,i}$ in terms of $\mathbf{A}$ with optimal $\mathbf{b}_{k,i}$ for each $\text{SLNR}_{k,i}$ and then to optimize $\mathbf{A}$ to maximize the sum SLNR.

A. Digital Precoder Design

With the assumption of channel state information at the transmitter (CSIT), for given $\mathbf{A}$, the optimal digital beamforming vector $\mathbf{b}_{k,i}$ for user $k^{(i)}$ at the i-th subcarrier is given by solving the following problem:

Problem 1:

$$\mathbf{b}_{k,i}^\star = \underset{\mathbf{b}_{k,i}}{\arg\max} \;\; \frac{\rho|\mathbf{h}_{k^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2}{\rho\sum_{p\neq k}|\mathbf{h}_{p^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2+1} \quad (17)$$
$$\text{s.t.} \;\; \mathbf{b}_{k,i}^H\mathbf{A}^H\mathbf{A}\mathbf{b}_{k,i} = 1.$$

By exploiting the constraint $\mathbf{b}_{k,i}^H\mathbf{A}^H\mathbf{A}\mathbf{b}_{k,i} = 1$, the $\text{SLNR}_{k,i}$ can be rewritten as

$$\begin{aligned}\text{SLNR}_{k,i} &= \frac{\rho|\mathbf{h}_{k^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2}{\rho\sum_{p\neq k}|\mathbf{h}_{p^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}|^2+\mathbf{b}_{k,i}^H\mathbf{A}^H\mathbf{A}\mathbf{b}_{k,i}} \\ &= \frac{\mathbf{b}_{k,i}^H\mathbf{A}^H\mathbf{h}_{k^{(i)},i}\mathbf{h}_{k^{(i)},i}^H\mathbf{A}\mathbf{b}_{k,i}}{\mathbf{b}_{k,i}^H\mathbf{A}^H\left(\sum_{p\neq k}\mathbf{h}_{p^{(i)},i}\mathbf{h}_{p^{(i)},i}^H+\mathbf{I}/\rho\right)\mathbf{A}\mathbf{b}_{k,i}}.\end{aligned} \quad (18)$$

Since $\mathbf{A}^H\mathbf{h}_{k^{(i)},i}\mathbf{h}_{k^{(i)},i}^H\mathbf{A}$ is a rank-one matrix, a closed-form solution to Problem 1 can be obtained based on (18) by generalized eigenvalue decomposition (GED) [11]. The optimal digital precoder is given by

$$\mathbf{b}_{k,i}^\star = \beta\left[\mathbf{A}^H\left(\sum_{p\neq k}\mathbf{h}_{p^{(i)},i}\mathbf{h}_{p^{(i)},i}^H+\mathbf{I}/\rho\right)\mathbf{A}\right]^{-1}\mathbf{A}^H\mathbf{h}_{k^{(i)},i},$$

where β is a constant to satisfy the power constraint, and the corresponding $\text{SLNR}_{k,i}$ is given by

$$\begin{aligned}&\text{SLNR}_{k,i} \\ &= \mathbf{h}_{k^{(i)},i}^H\mathbf{A}\left[\mathbf{A}^H\left(\sum_{p\neq k}\mathbf{h}_{p^{(i)},i}\mathbf{h}_{p^{(i)},i}^H+\mathbf{I}/\rho\right)\mathbf{A}\right]^{-1}\mathbf{A}^H\mathbf{h}_{k^{(i)},i} \\ &= \text{Tr}\left\{\mathbf{A}^H\mathbf{H}_{k,i}^N\mathbf{A}\left[\mathbf{A}^H\mathbf{H}_{k,i}^D\mathbf{A}\right]^{-1}\right\},\end{aligned} \quad (19)$$

where $\mathbf{H}_{k,i}^N = \mathbf{h}_{k^{(i)},i}\mathbf{h}_{k^{(i)},i}^H$ and $\mathbf{H}_{k,i}^D = \big(\sum_{p\neq k}\mathbf{h}_{p^{(i)},i}\mathbf{h}_{p^{(i)},i}^H + \mathbf{I}/\rho\big)$.

B. Analog Beamformer Design

In the previous subsection, the SLNR for each user with optimal digital precoding is expressed as a function of only the analog beamforming matrix $\mathbf{A}$. With this expression the optimal analog beamformer design problem can be formulated as follows:

Problem 2:

$$\mathbf{A}^\star = \underset{\mathbf{A}}{\arg\max} \quad \sum_{k=1}^{K}\sum_{i=1}^{N_f}\text{Tr}\left(\mathbf{A}^H\mathbf{H}_{k,i}^N\mathbf{A}[\mathbf{A}^H\mathbf{H}_{k,i}^D\mathbf{A}]^{-1}\right)$$
$$\text{s.t.} \;\; \mathbf{a}_r^H\mathbf{a}_r = 1 \text{ for } r = 1,\cdots,N_a. \quad (20)$$

If there were only one trace term in the cost function in (20), the problem would reduce to a *ratio trace problem* and would easily be solved by GED; the N_a columns of $\mathbf{A}$ would be given by the normalized generalized eigenvectors associated with the N_a largest generalized eigenvalues of the matrix pencil $(\mathbf{H}_{k,i}^N, \mathbf{H}_{k,i}^D)$ [12]. However, in Problem 2 we have multiple SLNR trace terms in the cost and hence the problem cannot be solved by simply applying GED. To tackle this problem, we derive a lower bound of the cost in (20) in form of a single trace quotient and maximize the derived lower bound.

Proposition 2: If $\mathbf{N}$ is an $N\times N$ rank-one positive semi-definite matrix and $\mathbf{D}$ is an $N\times N$ positive definite matrix. Then, the following inequality holds.

$$\text{Tr}\left(\mathbf{N}\mathbf{D}^{-1}\right) \geq \frac{\text{Tr}\left(\mathbf{N}\right)}{\text{Tr}\left(\mathbf{D}\right)}. \quad (21)$$

Proof:

$$\text{Tr}(\mathbf{N}\mathbf{D}^{-1}) \geq \sum_{r=1}^{N} \lambda_r(\mathbf{N})\lambda_{N-r+1}(\mathbf{D}^{-1}) \tag{22}$$

$$= \lambda_1(\mathbf{N})\lambda_N(\mathbf{D}^{-1}) = \frac{\lambda_1(\mathbf{N})}{\lambda_1(\mathbf{D})} \tag{23}$$

$$\geq \frac{\sum_{r=1}^{N} \lambda_r(\mathbf{N})}{\sum_{r=1}^{N} \lambda_r(\mathbf{D})} \tag{24}$$

$$= \frac{\text{Tr}(\mathbf{N})}{\text{Tr}(\mathbf{D})}, \tag{25}$$

where $\lambda_1(\mathbf{M}) \geq \lambda_2(\mathbf{M}) \geq \cdots \geq \lambda_N(\mathbf{M})$ indicate the N eigenvalues of an $N \times N$ matrix $\mathbf{M}$. Here, (22) is valid by an eigenvalue inequality for matrix product [13, Lemmas 1,2]. (23) is valid because $\mathbf{N}$ has rank one and $\lambda_N(\mathbf{D}^{-1}) = 1/\lambda_1(\mathbf{D})$. (24) holds since $\lambda_1(\mathbf{N}) = \sum_{r=1}^{N} \lambda_r(\mathbf{N})$ due to $\mathbf{N}$'s having rank one and $\lambda_1(\mathbf{D}) \leq \sum_{r=1}^{N} \lambda_r(\mathbf{D})$ due to the positive definiteness of $\mathbf{D}$. Finally, (25) holds because the trace of a matrix is the sum of its eigenvalues. ■

Applying Proposition 2 to the optimization cost in (20), we have a lower bound of the optimization cost given by

$$\sum_{k=1}^{K}\sum_{i=1}^{N_f} \text{Tr}\left(\mathbf{A}^H\mathbf{H}_{k,i}^N\mathbf{A}\left(\mathbf{A}^H\mathbf{H}_{k,i}^D\mathbf{A}\right)^{-1}\right)$$
$$\geq \sum_{k=1}^{K}\sum_{i=1}^{N_f} \frac{\text{Tr}\left(\mathbf{A}^H\mathbf{H}_{k,i}^N\mathbf{A}\right)}{\text{Tr}\left(\mathbf{A}^H\mathbf{H}_{k,i}^D\mathbf{A}\right)}$$
$$\geq \sum_{k=1}^{K}\sum_{i=1}^{N_f} \frac{\text{Tr}\left(\mathbf{A}^H\mathbf{H}_{k,i}^N\mathbf{A}\right)}{\sum_{k=1}^{K}\sum_{i=1}^{N_f}\text{Tr}\left(\mathbf{A}^H\mathbf{H}_{k,i}^D\mathbf{A}\right)} \tag{26}$$
$$= \frac{\text{Tr}\left(\mathbf{A}^H\left(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^N\right)\mathbf{A}\right)}{\text{Tr}\left(\mathbf{A}^H\left(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^D\right)\mathbf{A}\right)}, \tag{27}$$

where (26) holds since $\mathbf{H}_{k,i}^D$ is positive definite $\forall\ k, i$. Thus, the analog beamformer design problem can be reformulated as maximizing the lower bound of the sum SLNR:

Problem 3:

$$\mathbf{A}^* = \arg\max_{\mathbf{A}} \ \frac{\text{Tr}\left(\mathbf{A}^H\left(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^N\right)\mathbf{A}\right)}{\text{Tr}\left(\mathbf{A}^H\left(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^D\right)\mathbf{A}\right)}$$
$$\text{s.t.}\ \ \mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1, \cdots, N_a. \tag{28}$$

The above problem with a relaxed constraint $\sum_{r=1}^{N_a}\mathbf{a}_r^H\mathbf{a}_r = N_a$ instead of $\mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1, \cdots, N_a$ was solved in [9, Sec.V.]. However, with the stricter constraint of $\mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1, \cdots, N_a$, the solution in [9] is not applicable. One way to solve Problem 3 is to impose a stricter constraint of $\mathbf{A}^H\mathbf{A} = \mathbf{I}$ than $\mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1, \cdots, N_a$. Problem 3 with the new constraint $\mathbf{A}^H\mathbf{A} = \mathbf{I}$ is called a *trace ratio problem or trace quotient problem* and the exact solution of a trace ratio problem can be obtained by applying one of several existing numerical algorithms [10], [14]. Furthermore, Problem 3 can be approximated by the following *ratio trace problem*:

Problem 4:

$$\max_{\mathbf{A}} \ \text{Tr}\left\{\mathbf{A}^H\left(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^N\right)\mathbf{A} \cdot \left[\mathbf{A}^H\left(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^D\right)\mathbf{A}\right]^{-1}\right\} \tag{29}$$
$$\text{s.t.}\ \ \mathbf{a}_r^H\mathbf{a}_r = 1,\ \forall r = 1, \cdots, N_a.$$

This ratio trace problem was nicely solved in [12] by using GED, as already mentioned. Let $\mathbf{v}_i$ be the generalized eigenvector of the matrix pencil $(\sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^N, \sum_{k=1}^{K}\sum_{i=1}^{N_f}\mathbf{H}_{k,i}^D)$ associated with the i-th largest generalized eigenvalue. Then, the solution to Problem 4 is given by $\widehat{\mathbf{A}^*} = \left[\frac{\mathbf{v}_1}{||\mathbf{v}_1||}, \frac{\mathbf{v}_2}{||\mathbf{v}_2||}, \cdots, \frac{\mathbf{v}_{N_a}}{||\mathbf{v}_{N_a}||}\right]$. Note that the cost in (29) depends only on the subspace spanned by $\mathbf{A}$ [12]. The solution to Problem 3 with the relaxed constraint $\sum_{r=1}^{N_a}\mathbf{a}_r^H\mathbf{a}_r = N_a$ is given by $\eta\left[\mathbf{v}_1, \cdots, \mathbf{v}_{N_a}\right]$ for some constant η [9] .

IV. Numerical Results

We here provide some numerical result to evaluate the performance of our hybrid beamformer design algorithm presented in the previous section. We considered a MU-MISO-OFDM system with the number of BS transmit antennas $N_t = 64$, the number of RF chains $N_a = 8$, and the number of subcarriers $N_f = 128$. We set the number of users served at each subcarrier to be the same as the number of RF chains, i.e., $K = N_a = 8$. We assumed that the size of CP is $N_{cp} = N_f/8$ and used the wideband MISO channel model (6) with $D = N_{cp}$ and $L_s = 8$. The gain and AoD for the model (6) were independently generated according to $\alpha_{k^{(i)},m,l} \sim \mathcal{CN}(0, 1)$ and $\phi_{k^{(i)},m,l} \sim$ Unif$[-\pi/2, \pi/2]$. We set the noise to have zero mean and unit power $\sigma^2 = 1$.

Fig. 1 shows the sum rate performance of the proposed wideband hybrid beamformer design algorithm versus the signal-to-noise-ratio (SNR) defined as $\rho = P/(\sigma^2 N_a N_f)$. For a reference, we used a modified version of the narrow-band MU hybrid beamformer design method in [3]; we applied the method in [3] to the center subcarrier to design an analog beamformer and then, with the effective channel given by the product of the analog beamformer and the actual channel for each subcarrier, we designed a separate digital beamformer for each subcarrier based on ZF. We also considered the case in which the analog beamformer is designed based on the proposed method and the digital beamformer is designed separately with ZF based on the effective channel for each subcarrier. It is seen that the proposed analog and digital beamformer design method performs better than the other two methods. Note that there exists gain in designing a common analog beamformer by considering impacts on all subcarrier channels and there exists additional gain in designing digital beamformers based on SLNR over ZF at low SNR.

V. Conclusion

In this paper, we have considered analog-and-digital hybrid beamformer design for mmWave wideband MU-MISO-OFDM downlink systems, a common analog beamformer is used

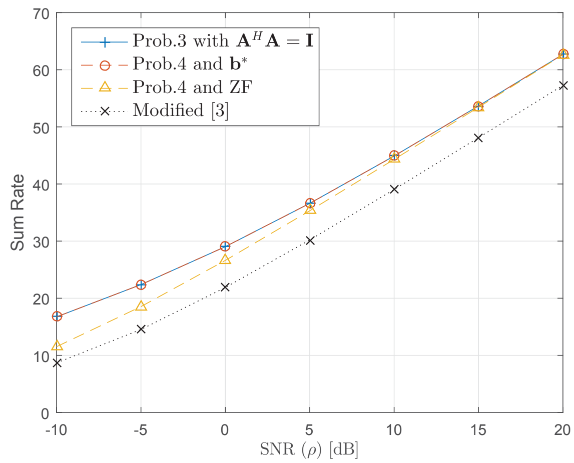

Fig. 1. Sum rate performance

for all subcarriers while a separate digital beamformer is applied to each subcarrier. Adopting the SLNR approach to this problem, we have derived the optimal digital beamformer and the corresponding SLNR for each user as functions of the analog beamformer. Based on this derived expression, we have formulated the common analog beamformer design problem as a trace quotient problem, which can efficiently be solved by existing algorithms. The derived algorithm is simple and provides an effective way to joint design of the analog beamformer and digital beamformers in wideband hybrid beamforming.

Appendix

$$
\begin{aligned}
&\mathbf{F}\mathbf{G}_{k(i)}(\mathbf{I}_{N_f}\otimes\mathbf{A})(\mathbf{F}^H\otimes\mathbf{I}_{N_a})\\
&=\mathbf{F}\mathbf{G}_{k(i)}(\mathbf{F}^H\otimes\mathbf{A}) && (30)\\
&=\mathbf{F}\mathbf{G}_{k(i)}\boldsymbol{\Pi}_1\boldsymbol{\Pi}_1^T(\mathbf{F}^H\otimes\mathbf{A}) && (31)\\
&=\mathbf{F}\left[\mathbf{G}_{k(i)}^{(1)}\ \cdots\ \mathbf{G}_{k(i)}^{(N_t)}\right]\begin{bmatrix}\mathbf{F}^H\otimes\boldsymbol{\alpha}_1^H\\ \vdots\\ \mathbf{F}^H\otimes\boldsymbol{\alpha}_{N_t}^H\end{bmatrix} && (32)\\
&=\mathbf{F}\sum_{n=1}^{N_t}\mathbf{G}_{k(i)}^{(n)}(\mathbf{F}^H\otimes\boldsymbol{\alpha}_n^H)\\
&=\mathbf{F}\sum_{n=1}^{N_t}\mathbf{G}_{k(i)}^{(n)}(\mathbf{F}^H\otimes\boldsymbol{\alpha}_n^H)\boldsymbol{\Pi}_2\boldsymbol{\Pi}_2^T && (33)\\
&=\mathbf{F}\sum_{n=1}^{N_t}\mathbf{G}_{k(i)}^{(n)}\left[a_{n,1}\mathbf{F}^H,\cdots,a_{n,N_a}\mathbf{F}^H\right]\boldsymbol{\Pi}_2^T && (34)\\
&=\left[\sum_n a_{n,1}\mathbf{F}\mathbf{G}_{k(i)}^{(n)}\mathbf{F}^H,\cdots,\sum_n a_{n,N_a}\mathbf{F}\mathbf{G}_{k(i)}^{(n)}\mathbf{F}^H\right]\boldsymbol{\Pi}_2^T.
\end{aligned}
$$

Here, (30) holds because $(\mathbf{A}\otimes\mathbf{B})(\mathbf{C}\otimes\mathbf{D}) = (\mathbf{AC})\otimes(\mathbf{BD})$. In (31), $\mathbf{I} = \boldsymbol{\Pi}_1\boldsymbol{\Pi}_2^T$ is inserted, where $\boldsymbol{\Pi}_1$ is the $N_fN_t\times N_fN_t$ column permutation matrix that converts a block circulant matrix to a row-wise concatenation of circulant matrices. That is, $\mathbf{G}_{k(i)}\boldsymbol{\Pi}_1 = \left[\mathbf{G}_{k(i)}^{(1)}\ \cdots\ \mathbf{G}_{k(i)}^{(N_t)}\right]$, where $\mathbf{G}_{k(i)}^{(n)}$ is an $N_f\times N_f$ circulant matrix composed of the columns $n, n+N_t, n+2N_t,\cdots, n+(N_f-1)N_t$ of $\mathbf{G}_{k(i)}$. $\boldsymbol{\Pi}_1^T(\mathbf{F}^H\otimes\mathbf{A})$ in (31) is expressed as the last term in (32), where $\boldsymbol{\alpha}_n^H = [a_{n,1}\ \cdots\ a_{n,N_a}]$ is the n-th row of $\mathbf{A}$, to yield (32). In (33), $\mathbf{I} = \boldsymbol{\Pi}_2\boldsymbol{\Pi}_2^T$ is inserted, where $\boldsymbol{\Pi}_2$ is the $N_fN_a\times N_fN_a$ column permutation matrix that converts $(\mathbf{F}^H\otimes\boldsymbol{\alpha}_n^H)\boldsymbol{\Pi}_2$ to $\left[a_{n,1}\mathbf{F}^H\ \cdots\ a_{n,N_a}\mathbf{F}^H\right]$.

Since $\mathbf{G}_{k(i)}^{(n)}$ is a circulant matrix, it can be diagonalized by DFT. Let $\mathbf{F}\mathbf{G}_{k(i)}^{(n)}\mathbf{F}^H = \boldsymbol{\Lambda}_{k(i)}^{(n)} = \mathrm{diag}(\lambda_{k(i),1}^{(n)},\ \ldots,\ \lambda_{k(i),N_f}^{(n)})$. Then, we have

$$
\begin{aligned}
&\mathbf{F}\mathbf{G}_{k(i)}(\mathbf{I}_{N_f}\otimes\mathbf{A})(\mathbf{F}^H\otimes\mathbf{I}_{N_a})\\
&=\left[\sum_{n=1}^{N_t}a_{n,1}\boldsymbol{\Lambda}_{k(i)}^{(n)},\cdots,\sum_{n=1}^{N_t}a_{n,N_a}\boldsymbol{\Lambda}_{k(i)}^{(n)}\right]\boldsymbol{\Pi}_2^T\\
&=\begin{bmatrix}\sum_{n=1}^{N_t}\lambda_{k(i),1}^{(n)}\boldsymbol{\alpha}_n^H & & \\ & \ddots & \\ & & \sum_{n=1}^{N_t}\lambda_{k(i),N_f}^{(n)}\boldsymbol{\alpha}_n^H\end{bmatrix}\\
&=\begin{bmatrix}[\lambda_{k(i),1}^{(1)}\cdots\lambda_{k(i),1}^{(N_t)}]\mathbf{A} & & \\ & \ddots & \\ & & [\lambda_{k(i),N_f}^{(1)}\cdots\lambda_{k(i),N_f}^{(N_t)}]\mathbf{A}\end{bmatrix}\\
&=\mathrm{diag}(\mathbf{h}_{k(i),1}^H\mathbf{A},\cdots,\mathbf{h}_{k(i),N_f}^H\mathbf{A}),
\end{aligned}
$$

where $\mathbf{h}_{k(i),j}^H := [\lambda_{k(i),j}^{(1)}\ \cdots\ \lambda_{k(i),j}^{(N_t)}]$ for $j = 1,\ldots,N_f$

References

[1] A. F. Molisch, V. V. Ratnam, S. Han, Z. Li, S. L. H. Nguyen, L. Li, and K. Haneda, "Hybrid beamforming for massive MIMO - A survey," *CoRR*, vol. abs/1609.05078, 2016.

[2] G. T. 36.897, "Study on Elevation Beamforming/Full-Dimension (FD) MIMO for LTE," July 2015.

[3] A. Alkhateeb, G. Leus, and R. W. Heath, "Limited feedback hybrid precoding for multi-user millimeter wave systems," *IEEE Transactions on Wireless Communications*, vol. 14, pp. 6481–6494, Nov 2015.

[4] F. Sohrabi and W. Yu, "Hybrid digital and analog beamforming design for large-scale antenna arrays," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, pp. 501–513, April 2016.

[5] X. Yu, J. Shen, J. Zhang, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave mimo systems," *IEEE Journal of Selected Topics in Signal Processing*, vol. 10, pp. 501–513, April 2016.

[6] C. Kim, T. Kim, and J. Y. Seol, "Multi-beam transmission diversity with hybrid beamforming for mimo-ofdm systems," in *2013 IEEE Globecom Workshops (GC Wkshps)*, pp. 61–65, Dec 2013.

[7] A. Alkhateeb and R. W. Heath, "Frequency selective hybrid precoding for limited feedback millimeter wave systems," *IEEE Transactions on Communications*, vol. 64, pp. 1801–1818, May 2016.

[8] S. Park, A. Alkhateeb, and R. W. H. Jr., "Dynamic subarrays for hybrid precoding in wideband mmwave MIMO systems," *CoRR*, vol. abs/1606.08405, 2016.

[9] M. Sadek, A. Tarighat, and A. H. Sayed, "A leakage-based precoding scheme for downlink multi-user MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1711 –1721, May 2007.

[10] D. Kim, G. Lee, and Y. Sung, "Two-stage beamformer design for massive mimo downlink by trace quotient formulation," *IEEE Transactions on Communications*, vol. 63, pp. 2200–2211, June 2015.

[11] J. So, D. Kim, Y. Lee, and Y. Sung, "Pilot signal design for massive mimo systems: A received signal-to-noise-ratio-based approach," *IEEE Signal Processing Letters*, vol. 22, pp. 549–553, May 2015.

[12] K. Fukunaga, *Introduction to Statistical Pattern Recognition*. Academic press, 2013.

[13] F. Zhang and Q. Zhang, "Eigenvalue inequalities for matrix product," *IEEE Transactions on Automatic Control*, vol. 51, pp. 1506–1509, Sept 2006.

[14] Y. Jia, F. Nie, and C. Zhang, "Trace ratio problem revisited," *IEEE Transactions on Neural Networks*, vol. 20, pp. 729–735, April 2009.