Coordinated Beamforming with Relaxed Zero Forcing

(Invited Paper)

G. Lee, J. Park and Y. Sung†
Dept. of Electrical Engineering
KAIST
Daejeon, Korea 305-701
Email: {gwlee@, jhp@ and ysung@ee.}kaist.ac.kr

M. Yukawa
Dept. Electric. Electron. Engineer.
Niigata University
Niigata, Japan 950-2181
Email: yukawa@eng.niigata-u.ac.jp

***Abstract*—In this paper, a new beam design paradigm for coordinated beamforming (CB) for current and future cellular networks is proposed based on a relaxed zero-forcing (RZF) constraint. In the conventional zero-forcing (ZF) CB, each co-operating transmitter designs its transmit beamforming matrix to null out interference to undesired receivers completely. In the proposed RZF paradigm, however, the ZF constraint is relaxed so that a tolerable amount of interference leakage to undesired receivers is allowed for the beam design at each transmitter. By relaxing the ZF constraint in such a way, transmitters have more degrees of freedom for their beam design to increase the overall data rate of the network. An efficient algorithm for the RZFCB is proposed based on a projected subgradient method. Numerical results show that the proposed RZFCB shows a noticeable gain over the conventional ZFCB.**

***Index Terms*—Coordinated beamforming, MIMO system, Zero-forcing, Interference channel, Projected subgradient method.**

I. Introduction

The necessity for high data rate and spectral efficiency is rapidly increasing for current and future cellular networks. To support the required high spectral efficiency, cellular networks should be densely populated and the cell size should become small. In this trend, cellular networks become interference-limited, and handling interference in the network is one of the most crucial problems of current and future cellular networks. Among many ways to handling the interference, multiple-input multiple-output (MIMO) antenna techniques and basestation cooperation are considered as the main key to the interference problem in current and future cellular networks. Indeed, the Third Generation Partnership Project (3GPP) Long-Term Evolution-Advanced (LTE-Advanced) considers the basestation cooperation and MIMO techniques to mitigate inter-cell interference under the name of Coordinated Multipoint (CoMP) [1] [2]. There are several candidate techniques for the basestation cooperation for the downlink [3], [4]. First, full joint processing based on the dirty paper coding (DPC) can be considered [5]. However, this method requires full data sharing in addition to channel state information (CSI) sharing among cooperating basestations, which reduces the practicalness of the method, and robust DPC techniques in-susceptible to CSI error have not been invented yet. Second, a simple but practical method of coordinated scheduling can be considered [6]. In this method, cooperating basestations take turns transmitting data, and consequently the served receiver experiences no inter-cell interference. However, there exists an inevitable loss in this method since other basestations than the transmitting one lose the opportunity to use the channel. Thus, coordinated beamforming (CB) is considered as a strong candidate compromising the complexity and performance of the basestation cooperation [2]. In the CB scheme, neither data nor full CSI is shared among cooperating basestations, but the cooperating basestations require local CSI only to design their beamforming matrices with the consideration of interference to undesired receivers. This local CSI sharing aspect of the scheme is especially desirable when inter-basestation communication is limited. Due to such desirable aspects, extensive research has been conducted on methods for designing beamforming matrices for the CB scheme. Up to now, most beam design methods for the CB scheme are based on ZF (or block diagonalization (BD)) [7]–[11]. In the ZF (or BD) scheme, the transmit beamforming matrix of a particular basestation is designed so that the basestation does not cause interference to other receivers served by other basestations. Such a beamforming matrix can easily be obtained by subspace decomposition of a matrix composed of MIMO channel matrices from the basestation to undesired receivers. Although the ZFCB eliminates inter-cell interference effectively, it reduces the degree of freedom for selecting the transmit beamforming matrix at each basestation; the transmit beamforming matrix should be designed so that its column space should be contained in the null space of the matrix composed of undesired channel matrices. When the row space of the desired channel matrix is not aligned with the above-mentioned null space, the rate experienced by the desired receiver is not high. Although such a loss can be compensated for by user scheduling [12], the required scheduling is complicated, and its effect is limited when the number of users in the cell is finite.

In this paper, we propose a method to alleviate the penalty associated with the ZFCB and to improve the rate performance of CB. The idea starts from a simple fact that the ZFCB overreacts to inter-cell interference. Most receivers (i.e., mobile stations) that are affected by inter-cell interference are cell-edge users, and thus there remains thermal noise even if

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the inter-cell interference is completely removed. Thus, it is unnecessary to completely eliminate the inter-cell interference, and it suffices to limit the inter-cell interference below a certain level lower than the thermal noise. Our approach to accomplishing this is based on relaxing the ZF constraint. In the proposed scheme, the inter-cell interference constraint is given by a quadratic constraint based on the Frobenius norm of a certain matrix, whereas the ZF constraint is given by a linear one. (This will become clear shortly in Section II.) The corresponding feasible set for our relaxed interference constraint is an ellipsoid that contains the feasible set of the ZF constraint. Thus, the feasible set for beamforming matrices is enlarged, and higher rates can be achieved. Under this new constraint, the problem of obtaining the optimal beamforming matrix is formulated as a constrained convex optimization, where the cost function is given by the rate for the desired receiver and the constraints are given by the transmit power and inter-cell interference level. An efficient solution to this optimization problem is proposed based on a projected sub-gradient method [13] [14], which guarantees convergence to the globally optimal point. Numerical results show that the proposed scheme for CB yields a noticeable gain over the ZFCB. The proposed scheme requires the same amount of CSI sharing as the ZFCB, and the complexity of the proposed scheme is not prohibitive. Therefore, the proposed scheme provides a better way to designing the transmit beamforming matrices for CB.

Notation and Organization

Standard notational conventions will be used in this paper. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For matrix $\mathbf{A}$, $\mathbf{A}^H$, $\|\mathbf{A}\|$, $\|\mathbf{A}\|_F$ and $\text{tr}(\mathbf{A})$ indicate the Hermitian transpose, 2-norm, Frobenius norm and trace of $\mathbf{A}$, respectively, and $\text{vec}(\mathbf{A})$ denotes the vector composed of the columns of $\mathbf{A}$. $\mathbf{I}_n$ stands for the identity matrix of size n (the subscript is omitted when unnecessary), and $\text{diag}(d_1, \cdots, d_n)$ denotes a diagonal matrix with diagonal elements $d_1, \cdots, d_n$. For matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \otimes \mathbf{B}$ represents the Kronecker product between $\mathbf{A}$ and $\mathbf{B}$, and $\mathbf{A} \geq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B}$ is positive semi-definite. $\mathbb{R}$ denotes the set of real numbers. $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is complex Gaussian-distributed with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$.

The rest of the paper is organized as follow. The system model and preliminary are provided in Section II. In Section III, the new beam design method for CB based on relaxed ZF is provided, and its performance is evaluated in Section IV, followed by conclusions in Section V.

II. Data Model and Preliminary

We consider a multi-cell MIMO network with K cooperating basestations, as illustrated in Fig. 1. Each basestation has N transmit antennas, and transmits data to a corresponding receiver with M receive antennas. We assume that $N \geq KM$ and that the receivers are in the cell boundary to experience inter-cell interference. Then, the received signal at receiver i associated with basestation (or transmitter) i is given by

$$\mathbf{y}_i = \mathbf{H}_{ii}\mathbf{V}_i\mathbf{s}_i + \sum_{j=1, j\neq i}^{K} \mathbf{H}_{ij}\mathbf{V}_j\mathbf{s}_j + \mathbf{n}_i, \quad i \in \{1, \cdots, K\}, \quad (1)$$

where $\mathbf{H}_{ij}$ is the $M \times N$ channel matrix from transmitter j to receiver i, $\mathbf{V}_j$ is the $N \times d$ transmit beamforming matrix at transmitter j, $\mathbf{s}_j$ is the $d \times 1$ transmit symbol vector at transmitter j drawn from $\mathcal{N}(\mathbf{0}, \frac{P}{d}\mathbf{I}_d)$, and $\mathbf{n}_i \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. (We normalized the noise variance to one for simplicity.) Here, P and d are the total transmit power and the number of data streams at each transmitter, respectively. (We will use the terms basestation and transmitter interchangeably in this paper.)

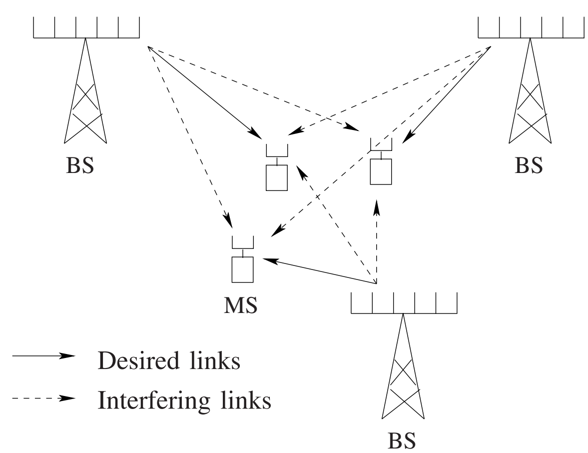

Fig. 1. A multi-cell MIMO network with three basestations and three served receivers (BS: basestation and MS: mobile station)

In the ZFCB, the interference to the undesired receivers is completely eliminated in the beam design [7]–[11]. For transmitter i, define a matrix $\tilde{\mathbf{H}}_i$ as the matrix composed of the undesired channel matrices:

$$\tilde{\mathbf{H}}_i := [\mathbf{H}_{1i}^T, \mathbf{H}_{2i}^T, \cdots, \mathbf{H}_{i-1,i}^T, \mathbf{H}_{i+1,i}^T, \cdots, \mathbf{H}_{Ki}^T]^T. \quad (2)$$

Then, the ZF condition at basestation i is given by

$$\tilde{\mathbf{H}}_i\mathbf{V}_i = \mathbf{0}, \quad (3)$$

which is a linear constraint on $\mathbf{V}_i$ since $\tilde{\mathbf{H}}_i$ is known. The size of $\tilde{\mathbf{H}}_i$ is $(K-1)M \times N$ and thus $\tilde{\mathbf{H}}_i$ has a non-trivial null space of $N - (K-1)M$ dimensions when $N \geq KM$. Thus, a beamforming matrix $\mathbf{V}_i$ for the ZFCB can be obtained based on the null space of $\tilde{\mathbf{H}}_i$, and the null space of $\tilde{\mathbf{H}}_i$ can easily be obtained by singular value decomposition (SVD) of $\tilde{\mathbf{H}}_i$ [8]:

$$\tilde{\mathbf{H}}_i = \tilde{\mathbf{U}}_i\tilde{\boldsymbol{\Sigma}}_i \begin{bmatrix} \tilde{\mathbf{V}}_{i0}^H \\ \tilde{\mathbf{V}}_{i1}^H \end{bmatrix}, \quad (4)$$

where $\tilde{\mathbf{U}}_i$ is a unitary matrix, and $\tilde{\mathbf{V}}_{i0}$ and $\tilde{\mathbf{V}}_{i1}$ are matrices of $N \times (K-1)M$ and $N \times (N - (K-1)M)$ sizes with orthonormal columns, respectively. Then, a ZF beamforming matrix is given by

$$\mathbf{V}_i = \tilde{\mathbf{V}}_{i1}\Theta_i, \quad (5)$$

where Θ_i is an arbitrary $(N-(K-1)M) \times d$ matrix ($(N-(K-1)M) \geq d$). The ZF beamforming matrix can further be optimized to yield maximum rate, and the optimal beamforming matrix for basestation i under the ZF condition is obtained by solving the following optimization problem:

$$\mathbf{V}_i^* = \arg\max_{\mathbf{V}_i} \log \left| \mathbf{I} + \frac{P}{d} \mathbf{H}_{ii} \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_{ii}^H \right|$$
$$\text{s.t.} \quad \tilde{\mathbf{H}}_i \mathbf{V}_i = \mathbf{0}, \quad \|\mathbf{V}_i\|_F^2 \leq d. \tag{6}$$

With $\mathbf{V}_i^*$ $(= \tilde{\mathbf{V}}_{i1}\Theta_i^*)$ for basestation i, $i = 1, \cdots, K$, the maximum sum rate under the ZFCB is given by

$$R_{sum}^{ZF} = \sum_{i=1}^{K} \log \left| \mathbf{I} + \frac{P}{d} \mathbf{H}_{ii} \mathbf{V}_i^* \mathbf{V}_i^{*H} \mathbf{H}_{ii}^H \right|. \tag{7}$$

Since $\mathbf{V}_i^*$ is chosen within the null space of $\tilde{\mathbf{H}}_i$, the ZF beamforming matrix is restricted in a $(N-(K-1)M)$-dimensional subspace. Thus, the rate performance of the ZFCB is not good when the row space of $\mathbf{H}_{ii}$, i.e., the subspace spanned by the row vectors of $\mathbf{H}_{ii}$, is not aligned with the null space of $\tilde{\mathbf{H}}_i$.

III. Beam Design for Rate Maximization under a Relaxed ZF Constraint

A. Problem formulation

Although the ZFCB provides an effective way to designing transmit beamforming matrices yielding no inter-cell interference, the scheme is not optimal from the perspective of data rate. For example, the sum rate optimality was studied for multiple-input single-output (MISO) interference channels in [15] [16], and it is shown that in the two-user case the transmit beam structure that achieves a Pareto-optimal point of the rate region is a linear combination of the ZF beam for the undesired receiver and the matched filtering beam for the desired receiver. However, an optimal strategy for the MIMO CB is not known yet, while the ZFCB is widely considered as a practical method. Here, we propose a new beam design paradigm for the MIMO CB by relaxing* the ZF constraint but still by limiting the inter-cell interference below a certain level, to have a larger feasible set for beam design and thus to have a larger sum rate than the ZFCB method. The new RZF constraint that we adopt is given by

$$\left\| \tilde{\mathbf{H}}_i \mathbf{V}_i \right\|_F^2 \leq \epsilon, \quad \forall\, i, \tag{8}$$

for some $\epsilon \geq 0$. When $\epsilon = 0$, the condition reduces to ZF. When $\epsilon > 0$, on the other hand, the ZF constraint is relaxed to include more beamforming matrices $\mathbf{V}_i$ satisfying the constraint. This is the reason why we refer to the constraint (8) as RZF. With this new interference constraint, the CB beam design problem is formulated as follows.

*Such a relaxation of the ZF constraint was recently considered in adaptive beamforming in [17]. It was shown that the relaxation method was very effective to improve the system performance.

Problem 1 (Beam design with RZF):

$$\max_{\{\mathbf{V}_i\}} \sum_{i=1}^{K} \log \left| \mathbf{I}_M + \frac{P}{d} (\mathbf{I} + \mathbf{B}_i)^{-1} \mathbf{H}_{ii} \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_{ii}^H \right| \tag{9}$$

such that (s.t.)

$$\text{(C.1)} \quad \|\tilde{\mathbf{H}}_i \mathbf{V}_i\|_F^2 \leq \frac{d}{P} \frac{\epsilon'}{K-1} = \epsilon, \tag{10}$$
$$\text{(C.2)} \quad \|\mathbf{V}_i\|_F^2 \leq d, \tag{11}$$

where $\mathbf{B}_i = \frac{P}{d} \sum_{j \neq i} \mathbf{H}_{ij} \mathbf{V}_j \mathbf{V}_j^H \mathbf{H}_{ij}^H$.

Note that in (9) the interference from other basestations is incorporated in the rate formula by the term $\mathbf{B}_i$ capturing the residual inter-cell interference under the RZF constraint. (10) is the RZF interference constraint, and (11) is the transmit power constraint. Here, the total tolerable interference power at each receiver is assumed to be ϵ'. The term d/P in the right-handed side (RHS) of (10) is due to the term $\mathbf{s}_j$ in the data model (1), and the term $1/(K-1)$ of the RHS of (10) is because we have $K-1$ interfering basestations. Note that $||\tilde{\mathbf{H}}_i \mathbf{V}_i||_F^2$ is the interference caused to all the undesired receivers by basestation i. By limiting this total interference below a certain level, we can make the interference caused to each of the undesired receivers below that level, i.e., $||\mathbf{H}_{ki} \mathbf{V}_i||_F^2 \leq ||\tilde{\mathbf{H}}_i \mathbf{V}_i||_F^2 \leq \epsilon$ for all $k \neq i$.

Problem 1 is a difficult problem, and requires central processing to be solved since the rate of one user is dependent on those of others through the interference covariance terms $\{\mathbf{B}_i\}$. However, our RZF constraint makes the problem simple. Note that under the RZF constraint the interference power from other basestations is upper bounded because of (8), i.e.,

$$\sum_{j \neq i} \frac{P}{d} \|\mathbf{H}_{ij} \mathbf{V}_j\|_F^2 = \text{tr}(\mathbf{B}_i) \leq \epsilon', \tag{12}$$

which implies

$$\mathbf{B}_i \leq \epsilon' \mathbf{I}. \tag{13}$$

The term inside the summation in (9) is the rate formula for a MIMO channel in which the effective noise has the covariance matrix $\mathbf{I} + \mathbf{B}_i$ with a power constraint tr$(\mathbf{I} + \mathbf{B}_i) \leq M + \epsilon'$. By Theorem 1 of [18], a lower bound on the rate is given by

$$\sum_{i=1}^{K} \log \left| \mathbf{I}_M + \frac{1}{1+\epsilon'} \frac{P}{d} \mathbf{H}_{ii} \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_{ii}^H \right|, \tag{14}$$

based on the fact that independent Gaussian noise is worst for the rate under a given noise power constraint. This lower bound can be used for the beam design. Note in (14) that the inter-basestation dependency is removed and the beam design can be performed at each basestation separately based on the same amount of CSI as the ZFCB. Based on the lower bound (14), the RZFCB problem is now formulated as follows.

Problem 2 (Localized beam design with RZF):

$$\min_{\mathbf{V}_i} \phi_i(\mathbf{V}_i) = \min_{\mathbf{V}_i} -\log \left| \mathbf{I}_M + \frac{1}{1+\epsilon'} \frac{P}{d} \mathbf{H}_{ii} \mathbf{V}_i \mathbf{V}_i^H \mathbf{H}_{ii}^H \right| \tag{15}$$

s.t.

$$\text{(C.1)} \quad \|\tilde{\mathbf{H}}_i\mathbf{V}_i\|_F^2 \leq \frac{\epsilon'}{P(K-1)/d} = \epsilon, \tag{16}$$

$$\text{(C.2)} \quad \|\mathbf{V}_i\|_F^2 \leq d, \tag{17}$$

for each $i \in \{1, \cdots, K\}$.

Note that Problem 2 is a constrained convex problem since the log determinant is a concave function and the constraints sets are convex.

B. The beams design algorithm based on a projected subgradient method

Our solution to Problem 2 for the RZFCB is based on the following theorem by Polyak.

Theorem 1 (The projected subgradient method (PSM) [13]): Given a convex and continuous (not necessarily smooth) function $\phi : \mathcal{H} \to \mathbb{R}$ and a nonempty closed convex set $C \subset \mathcal{H}$, where $\mathcal{H}$ is a Hilbert space, assume that $\inf \phi(C) := \inf\{\phi(u) : u \in C\}$ is known, i.e., let without loss of generality $\inf \phi(C) = 0$. Let $\phi'(v)$ denote a subgradient of ϕ at a point $v \in \mathcal{H}$. Assume also that there exists a point $\hat{u} \in C$ with $\phi(\hat{u}) = 0$, and $||\phi'(u)|| \leq c$ on $\{u \in C : ||u - \hat{u}|| \leq ||\hat{u} - u_0||\}$ for some $c > 0$ and an arbitrary fixed $u_0 \in C$. Then, the following algorithm generates a sequence $\{u_n, n \geq 0\}$ s.t. $\lim_{n\to\infty} \phi(u_n) = 0$, and $\{u_n, n \geq 0\}$ converges weakly to some minimum point of ϕ over C: $\forall\ n \geq 0$,

$$u_{n+1} = \begin{cases} P_C\left(u_n - \lambda_n \frac{\phi(u_n)}{||\phi'(u_n)||^2}\phi'(u_n)\right), & \text{if } \phi'(u_n) \neq 0, \\ u_n, & \text{otherwise,} \end{cases} \tag{18}$$

where $\lambda_n \in [\varepsilon_1, 2 - \varepsilon_2]$ for some $\varepsilon_1, \varepsilon_2 > 0$. The mapping $P_C : \mathcal{H} \to C$ denotes the metric projection onto C, i.e., $||P_C(u) - u|| \leq ||v - u||, \ \ \forall\ v \in C$.

The projection in (18) is composed of two projections: the first is a subgradient projection and the second is a metric projection. Now, the application of the PSM to Problem 2 is straightforward. The convex cost function is given by (15) and the convex feasible set C is given by the intersection of two constraint sets generated by (C.1) and (C.2). Based on the properties of determinant [19], the gradient of the cost function is given by

$$\phi_i'(\mathbf{V}_i) = \frac{1}{1+\epsilon'}\frac{P}{d}\mathbf{H}_{ii}^H\mathbf{C}_i^{-1}\mathbf{H}_{ii}\mathbf{V}_i \tag{19}$$

where $\mathbf{C}_i = \mathbf{I}_M + \frac{P}{d}\mathbf{H}_{ii}\mathbf{V}_i\mathbf{V}_i^H\mathbf{H}_{ii}^H$. The constraints (C.1) and (C.2) can be rewritten in vector form respectively as

$$\|\breve{\mathbf{H}}_i\mathbf{v}_i\|^2 \leq \epsilon \ \ \text{and} \ \ \|\mathbf{v}_i\|^2 \leq d, \tag{20}$$

where $\mathbf{v}_i = \text{vec}(\mathbf{V}_i)$ and $\breve{\mathbf{H}}_i := \mathbf{I}_M \otimes \tilde{\mathbf{H}}_i$ $(\in \mathbb{C}^{(K-1)M^2 \times MN})$, and the corresponding feasible sets are given by

$$E_i := \{\mathbf{v} \in \mathbb{C}^{MN} : \mathbf{v}^H\mathbf{Q}_i\mathbf{v} \leq \epsilon\}, \quad \text{where } \mathbf{Q}_i = \breve{\mathbf{H}}_i^H\breve{\mathbf{H}}_i,$$
$$B_i := \{\mathbf{v} \in \mathbb{C}^{MN} : \|\mathbf{v}\|^2 \leq d\}.$$

The feasible set E_i for (C.1) is an ellipsoid and B_i for (C.2) is a sphere. Consequently, their intersection $C := E_i \cap B_i$ is also convex since the intersection of convex sets is convex [20]. The metric projection onto the intersection C does not have a closed-form expression, but it can be approximated by successive convex projections [21]. That is, we first project an initial point onto the ellipsoid E_i, which can easily be done by a known method like the one in [22], and then project the image of the first projection on the ball B_i, which is done as

$$P_{B_i}(\mathbf{v}) = \begin{cases} \mathbf{v}, & \text{if } ||\mathbf{v}||^2 \leq d, \\ \frac{\sqrt{d}\mathbf{v}}{||\mathbf{v}||}, & \text{otherwise,} \end{cases} \tag{21}$$

The successive projections onto convex sets do not guarantee convergence to the metric projection point of the original vector onto the intersection of the two convex feasible sets, but the resulting point resides in the intersection [21]. Based on recent results by Slavakis et al. [14], however, the convergence of the proposed algorithm based on the successive projections to the globally optimal point can be shown. The proposed algorithm for the RZFCB is summarized in the following table.

Algorithm 1

For each user,
0. initialize $\mathbf{V}_i$ as a ZF beamforming matrix;
1. compute $\phi'(\mathbf{V}_i)$ using (19);
2. perform the subgradient projection of $\mathbf{V}_i$ using (18);
3. perform successive metric projections of $\mathbf{V}_i$ onto E_i and then onto B_i;
4. go to Step 1 and repeat until the relative difference of $\phi_i(\mathbf{V}_i)$ is less than a predetermined threshold.

IV. Numerical Results

In this section, we provide some numerical results to evaluate the proposed RZFCB method. We considered two multi-cell scenarios $(K, M, N) = (3, 2, 6)$ and $(K, M, N) = (5, 2, 10)$. The channel elements for $\mathbf{H}_{ij}$ were generated independently and identically-distributedly (i.i.d.) from $\mathcal{N}(0, 1)$. The noise variance σ^2 was set to one, as explained already. We considered three values, σ^2, $10^{-1}\sigma^2$ and $10^{-2}\sigma^2$, for the relaxation parameter ϵ'. The step size parameter λ_n for the PSM was chosen as $\lambda_n = 0.03$ for all n. The signal-to-noise ratio (SNR) is defined as P/σ^2. Fig. 2 and Fig. 3 show the sum rate performance of the ZFCB and RZFCB methods averaged over 50 independent channel realizations for $(K, M, N) = (3, 2, 6)$ and $(K, M, N) = (5, 2, 10)$, respectively. (7) is used for the rate for the ZFCB. It is seen that the proposed RZFCB method outperforms the ZFCB for all SNR. The gain of the RZFCB over the ZFCB at low SNR is large, and this large gain at low SNR is especially important because most cell-edge receivers operate in the low SNR regime. (As expected, the gain diminishes as SNR increases since ZF is optimal at sufficiently high SNR.)

V. Conclusion

We have considered the beam design problem for coordinated beamforming. Based on a new relaxed zero-forcing

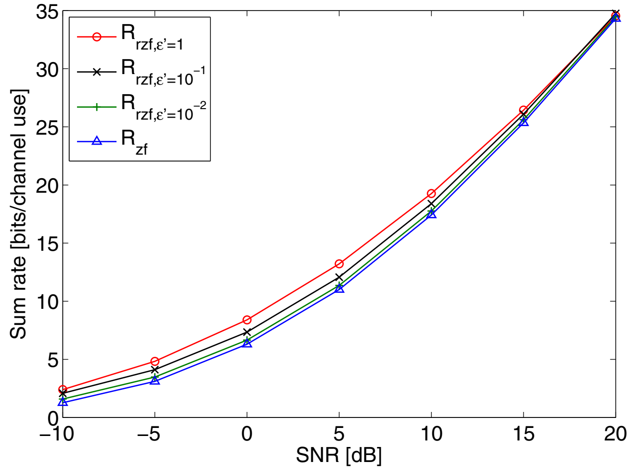

Fig. 2. Sum rate: $K = 3$, $M = 2$ and $N = 6$

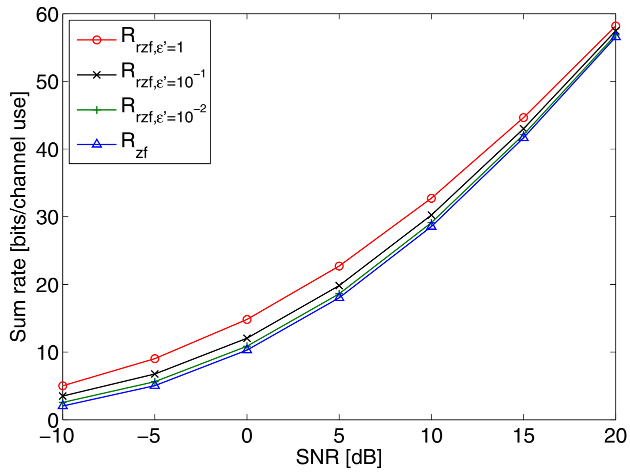

Fig. 3. Sum rate: $K = 5$, $M = 2$ and $N = 10$

constraint, we have proposed a new beam design paradigm for coordinated beamforming. We have also proposed an efficient algorithm based on a projected subgradient method to implement this new design paradigm. We have provided numerical results to validate our new design paradigm. The numerical results show that the proposed scheme outperforms the conventional zero-forcing coordinated beamforming method and, especially, the gain of the proposed method over the conventional zero-forcing method is large at low SNR. The complexity of the proposed method is not prohibitive, and the proposed method requires the same amount of channel information as the conventional zero-forcing method. Thus, the proposed method provides a better way to designing transmit beamforming matrices for coordinated beamforming for current and future cellular networks.

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