THE BAHADUR EFFICIENCY FOR ENERGY DETECTION OF STATIONARY GAUSSIAN PROCESSES

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ABSTRACT

In this paper, the performance and optimization of energy detection of stationary Gaussian signals are considered. Based on the Bahadur asymptotic relative efficiency, the performance of energy detection relative to optimal detection is compared, and the optimal threshold for energy detection is derived. It is shown that the optimal threshold for optimal detection is not optimal for energy detection, and an integral equation for determining the optimal threshold for energy detection is provided. A numerical example of the detection of equi-correlated signals is provided, and the numerical result validates our asymptotic analysis in the finite sample regime.

Index Terms- Energy detection, Bahadur efficiency, asymptotic relative efficiency, error exponent, large deviation principles

1. INTRODUCTION

Due to recent interest in cognitive radio communications, signal detection has gained a renewed interest. In cognitive radio communications, secondary users should detect the transmission of a primary user. Since the exact statistics of the primary signal are not available for secondary users, various robust techniques for the detection of unknown signals have been considered for this problem. Among them, the energy detection has drawn much interest due to its simplicity [1–5]. Typically, the detection of an unknown signal is modeled as the problem of detection of a Gaussian signal in Gaussian noise, which is a classical problem in detection theory [6]. In this case, the optimal detector is given by a quadratic detector under both Bayesian and Neyman-Pearson frameworks [6, p.7, p.24], and the simple energy detection is not optimal in general. Thus, the analysis of the loss or limitation of energy detection compared with optimal detection has been performed based on several measures. For example, Digham et al. derived the ROC region of the energy detector for various signal models [2], and Tandra et al. examined the limitation of energy detection with noise variance uncertainty [3]. In particular, the analysis of performance loss due to signal correlation is a difficult problem since the exact error probability of the energy detection of a Gaussian signal with correlation is not available. Thus, several researchers resort to asymptotic techniques to investigate this problem. For example, Lim et al. approached the problem by using the Pitman asymptotic relative efficiency (ARE), or equivalently, generalized signal-to-noise ratio, for the energy detection under a FIR channel model [4]. A similar criterion was employed to design a linear-quadratic fusion rule in a cooperative sensing environment in [7]. While the Pitman ARE provides a meaningful performance comparison in the low signal-to-noise ratio (SNR) regime, it is not a proper metric when the sample SNR is reasonably large. Thus, in this paper, we investigate the performance loss of energy detection based on the Bahadur ARE [8]. The ARE compares the number of samples for two detectors required to yield the same asymptotic performance. Under the Bahadur framework, the sample SNR is fixed, the sample size for the problem increases and the error probability decreases (typically exponentially), whereas under the Pitman framework the sample SNR is renormalized and decreasing so that the error probability does not decay. Based on large deviations principle (LDP) [9, 10], under the Bayesian framework we derive the Bahadur ARE for the energy detection by applying the Gärtner-Ellis theorem to a properly defined mismatched test statistic, as in [11], and obtain the optimal design for energy detection (i.e., obtain the optimal threshold, the unique design variable for this problem). We prove that the optimal threshold for optimal detection is not optimal for energy detection and that the Bahadur ARE is maximized when the threshold is designed to satisfy an equalizer rule; an integral equation for optimal threshold is provided.

The remainder of the paper is organized as follows. In Section 2, we provide the signal model and the problem formulation. In Section 3, we briefly review relevant results from LDP, investigate the optimal design for the threshold to maximize the Bahadur ARE, and provide a numerical example to validate our analysis. Finally, conclusion is in Section 4.

2. DATA MODEL AND PRELIMINARY

The detection problem that we consider in this paper is given by

$$\begin{array}{lll} \mathcal{H}_0: & y[i] = w[i], & i = 1, \ldots n, \\ \mathcal{H}_1: & y[i] = \theta r[i] + w[i], & i = 1, \ldots n, \end{array} \tag{1}$$

where $\{r[i]\}$ is a zero-mean stationary Gaussian process, and $\{w[i]\}$ is an independent and identically-distributed (i.i.d.) zero-mean Gaussian noise process, which is assumed to be independent of $\{r[i]\}$. Since $\{r[i]\}$ is stationary, its autocorrelation sequence and spectral density function are well defined and given by

$$\gamma_k \triangleq \mathbb{E}\{r[i]r[i-k]\} \text{ and } S_r(\omega) = \sum_{k=-\infty}^{\infty} \gamma_k e^{-j\omega k}, \tag{2}$$

respectively. We assume that $\{r[i]\}$ is scaled to have unit variance, i.e., $\gamma_0 = \mathbb{E}\{r[i]^2\} = 1$, and the signal amplitude is captured in the parameter θ. Then, the signal-to-noise ratio (SNR) is given by $\text{SNR} = \theta^2/\sigma^2$. The problem (1) can be rewritten in vector form as

$$\begin{array}{ll} \mathcal{H}_0: & \mathbf{y}_n \sim p_{0,n} = \mathcal{N}(0, \boldsymbol{\Sigma}_0), \\ \mathcal{H}_1: & \mathbf{y}_n \sim p_{1,n} = \mathcal{N}(0, \boldsymbol{\Sigma}_1), \end{array} \tag{3}$$

where $\mathbf{y}_n \triangleq [y[1], y[2], \cdots, y[i]]^T$, $\mathbf{r}_n \triangleq [r[1], r[2], \cdots, r[i]]^T$, $\mathbf{w}_n \triangleq [w[1], w[2], \cdots, w[i]]^T$, $\boldsymbol{\Sigma}_0 = \sigma^2 \mathbf{I}_n$, and $\boldsymbol{\Sigma}_1 = \theta^2 \boldsymbol{\Sigma}_r +$

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$\sigma^2\mathbf{I}_n$. Here, $\mathbf{\Sigma}_r$ is the covariance matrix of $\{r[i]\}$, given by $[\mathbf{\Sigma}_r]_{ij} = \gamma_{|i-j|}$.

It is known that the optimal detector that minimizes the Bayesian error probability is a log-likelihood ratio (LLR) detector: [6]

$$T_{n,opt} \triangleq \frac{1}{n}\log\left(\frac{p(\mathbf{y}_n|\mathcal{H}_1)}{p(\mathbf{y}_n|\mathcal{H}_0)}\right) \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \frac{1}{n}\log\frac{\pi_0}{\pi_1} =: \tau_{opt}, \quad (4)$$

where $p(\mathbf{y}_n|\mathcal{H}_i)$ and π_i are the probability density and prior probability for hypothesis i, respectively. Note that *for any prior probabilities π_0 and π_1, the asymptotically optimal threshold is zero!* The test statistic in (4) reduces to $\mathbf{y}_n^T\left(\sigma^2\mathbf{I}+\theta^2\mathbf{\Sigma}_r\right)^{-1}\mathbf{\Sigma}_r\mathbf{y}_n$, and computing this quantity requires $\mathcal{O}(n^2)$ complexity and also requires the receiver to know $\mathbf{\Sigma}_r$, which is not available in many cases. Thus, when the receiver has limited computation capability and little knowledge about signal statistics, an energy detector based on the energy statistic $\mathbf{y}_n^T\mathbf{y}_n = \sum_{i=1}^n y[i]^2$ is widely used. For later development, it is useful to view the energy detector as an equivalent LLR detector for a mismatched hypothesis detection problem, given by

$$\begin{array}{ll} \mathcal{H}_0: & \mathbf{y}_n \sim p_{0,n} = \mathcal{N}(0,\mathbf{\Sigma}_0), \\ \tilde{\mathcal{H}}_1: & \mathbf{y}_n \sim p_{1,n} = \mathcal{N}(0,\tilde{\mathbf{\Sigma}}_1), \end{array} \quad (5)$$

where

$$\mathbf{\Sigma}_0 = \sigma^2\mathbf{I}_n \quad \text{and} \quad \tilde{\mathbf{\Sigma}}_1 = (\sigma^2+\theta^2)\mathbf{I}_n. \quad (6)$$

Then, the equivalent test statistic is given by

$$T_{n,ed} \triangleq \frac{1}{n}\log\left(\frac{p(\mathbf{y}_n|\tilde{\mathcal{H}}_1)}{p(\mathbf{y}_n|\mathcal{H}_0)}\right) \gtrless^{\mathcal{H}_1}_{\mathcal{H}_0} \tau_{ed}, \quad (7)$$

which is nothing but the energy statistic. In the following section, we will examine the asymptotic performance of the energy detection compared with the optimal detection based on the Bahadur asymptotic relative efficiency (ARE).

3. THE BAHADUR EFFICIENCY OF ENERGY DETECTION

For the problem (1), both the optimal and energy detectors have exponentially decreasing error probability as the sample size n increases. That is, $P_{\delta_i} \sim C_i\exp(-nE_i)$ for some constant C_i, where E_i is the error exponent of detector δ_i. The Bahadur ARE ARE_{δ_1,δ_2} of detector δ_1 with respect to (w.r.t.) detector δ_2 is defined as

$$ARE_{\delta_1,\delta_2} = \frac{E_1}{E_2}.$$

Note that the Bahadur ARE compares the number of samples required to yield the same (asymptotic) performance for the two detectors (ignoring constants C_1 and C_2). Thus, when the Bayesian error exponents of the two detectors are known, we can assess the loss of the energy detector compared with the optimal detector. From here on, we will derive the Bahadur ARE of the energy detector w.r.t. the optimal detector based on the large deviations principle.

Here, we briefly review the fundamental theorem of LDP, which explains the asymptotic behavior of a sequence of random variables. Let $\{T_n\}$ be a sequence of random variables and let $\Lambda(u)$ be its asymptotic cumulant generating function (CGF), i.e.,

$$\Lambda(u) \triangleq \lim_{n\to\infty}\frac{1}{n}\log\mathbb{E}[\exp(nuT_n)]. \quad (8)$$

($\Lambda(u)$ can easily be verified to be convex.) Then, the asymptotic behavior of the tail probability of $\{T_n\}$ is given by the following theorem.

Theorem 1 (Gärtner-Ellis [10]) *Assume that limit* (8) *exists as an extended real number and origin belongs to the interior point of $\{u \in \mathbb{R} : \Lambda(u) < \infty\}$. Then the following holds:*

(i) For any closed set F,

$$\limsup_{n\to\infty}\frac{1}{n}\log\mathbb{P}(T_n \in F) \le -\inf_{z\in F}\Lambda^*(z). \quad (9)$$

(ii) For any open set G,

$$\liminf_{n\to\infty}\frac{1}{n}\log\mathbb{P}(T_n \in G) \ge -\inf_{z\in G}\Lambda^*(z), \quad (10)$$

where $\Lambda^*(z)$ refers the Fenchel-Legendre transform of $\Lambda(u)$, i.e.,

$$\Lambda^*(z) \triangleq \sup_{u\in\mathbb{R}}(zu - \Lambda(u)). \quad (11)$$

In addition, we need the following theorem for the asymptotic distribution of the eigenvalues of a Toeplitz covariance matrix.

Theorem 2 (Toeplitz distribution theorem [12]) *Let $\{\lambda_i^{(n)}\}$ be the eigenvalues of a Toeplitz covariance matrix $\mathbf{\Sigma}_n$ of a stationary process $\{y[i]\}$ with spectrum $S_y(e^{j\omega})$ having finite lower and upper bounds. Then,*

$$\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n h(\lambda_i^{(n)}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} h(S_y(e^{j\omega}))d\omega \quad (12)$$

for any continuous function $h(\cdot)$.

3.1. The Error Exponents of the Detectors

The Bayesian error exponent of the optimal detector is given by the Chernoff information: [13],

$$\begin{aligned} E_{opt} &= -\min_{u\in\mathbb{R}}\frac{1}{4\pi}\int_{-\pi}^{\pi}\left[\log\left(\frac{u}{S_1(\omega)}+\frac{1-u}{S_0(\omega)}\right)\right. \\ &\qquad \left. +u\log(S_1(\omega)) + (1-u)\log(S_0(\omega))\right]d\omega \quad (13)\\ &= -\min_{u\in\mathbb{R}}\frac{1}{4\pi}\int_{-\pi}^{\pi}\left[\log\left(1+(1-u)\mathrm{SNR}S_r(\omega)\right)\right. \\ &\qquad \left. +(u-1)\log(1+\mathrm{SNR}S_r(\omega))\right]d\omega. \quad (14) \end{aligned}$$

For the energy detector, we use the mismatched statistic $T_{n,ed}$ in (8) to obtain the asymptotic CGF under the true underlying distributions.

$$\begin{aligned} \Lambda_{ed,0}(u) &\triangleq \lim_{n\to\infty}\frac{1}{n}\log\mathbb{E}[\exp(nuT_{n,ed}|\mathcal{H}_0)] \\ &= \lim_{n\to\infty}\frac{1}{n}\log\frac{\left|\left(u\tilde{\mathbf{\Sigma}}_1^{-1}+(1-u)\mathbf{\Sigma}_0^{-1}\right)^{-1}\right|^{1/2}}{|\tilde{\mathbf{\Sigma}}_1|^{u/2}|\mathbf{\Sigma}_0|^{(1-u)/2}} \quad (15)\\ \Lambda_{ed,1}(u) &\triangleq \lim_{n\to\infty}\frac{1}{n}\log\mathbb{E}[\exp(nuT_{n,ed}|\mathcal{H}_1)] \\ &= \lim_{n\to\infty}\frac{1}{n}\log\frac{\left|\left(\mathbf{\Sigma}_1^{-1}+u\tilde{\mathbf{\Sigma}}_1^{-1}-u\mathbf{\Sigma}_0^{-1}\right)^{-1}\right|^{1/2}}{|\mathbf{\Sigma}_1|^{1/2}|\tilde{\mathbf{\Sigma}}_1|^{u/2}|\mathbf{\Sigma}_0|^{-u/2}} \end{aligned} \quad (16)$$

Now, by applying Theorem 2, we have

$$\Lambda_{0,ed}(u) = -\frac{1}{4\pi}\int_{-\pi}^{\pi}\Big[\log\left(\frac{u}{\tilde{S}_1(\omega)} + \frac{1-u}{S_0(\omega)}\right) + u\log(\tilde{S}_1(\omega)) + (1-u)\log(S_0(\omega))\Big]\,d\omega, \quad (17)$$

$$\Lambda_{1,ed}(u) = -\frac{1}{4\pi}\int_{-\pi}^{\pi}\Big[\log\left(\frac{1}{S_1(\omega)} + \frac{u}{\tilde{S}_1(\omega)} - \frac{u}{S_0(\omega)}\right) + \log(S_1(\omega)) + u\log(\tilde{S}_1(\omega)) - u\log(S_0(\omega))\Big]\,d\omega, \quad (18)$$

where $S_0(\omega) = \sigma^2$, $S_1(\omega) = \sigma^2 + \theta^2 S_r(\omega)$, and $\tilde{S}_1(\omega) = \sigma^2 + \theta^2$ are the spectral density functions of the signal under $\mathcal{H}_0$, $\mathcal{H}_1$, and $\tilde{\mathcal{H}}_1$, respectively.

Now, the exponents for the false alarm and miss detection probabilities of the energy detector are given by Theorem 1. If the threshold τ_{ed} for the test statistics $T_{n,ed}$ satisfies the condition $\frac{d}{du}\Lambda_{0,ed}(0) \le \tau_{ed} \le \frac{d}{du}\Lambda_{1,ed}(0)$, by Theorem 1, we have

$$\lim_{n\to\infty}\log P_F(n) = \lim_{n\to\infty}\log Pr(T_{n,ed} > \tau_{ed}|\mathcal{H}_0) = -\inf_{z>\tau_{ed}}\Lambda^*_{0,ed}(z) = -\Lambda^*_{0,ed}(\tau_{ed}), \quad (19)$$

$$\lim_{n\to\infty}\log P_M(n) = \lim_{n\to\infty}\log Pr(T_{n,ed} < \tau_{ed}|\mathcal{H}_1) = -\inf_{z<\tau_{ed}}\Lambda^*_{1,ed}(z) = -\Lambda^*_{1,ed}(\tau_{ed}), \quad (20)$$

where $\Lambda^*_{0,ed}(z)$ and $\Lambda^*_{1,ed}(z)$ are the Fenchel-Legendre transforms of $\Lambda_{0,ed}(u)$ and $\Lambda_{1,ed}(u)$, respectively. Since the Bayesian error probability is given by

$$P_{B,ed}(n,\tau_{ed}) = \pi_0 P_F(n) + \pi_1 P_M(n), \quad (21)$$

we have

$$E_{ed} = -\min\{\Lambda^*_{0,ed}(\tau_{ed}), \Lambda^*_{1,ed}(\tau_{ed})\} \quad (22)$$

Here, one could have used the asymptotically optimal threshold $\tau_{opt} = 0$ for the optimal detector blindly for the energy detector. However, this choice is not optimal, and the asymptotically optimal design for the energy detector is given by the following theorem.

Theorem 3 *The maximum error exponent E_{ed} for the energy detector is achieved if the threshold τ_{ed} satisfy the following equalizer rule:*

$$\Lambda^*_{0,ed}(\tau_{ed}) = \Lambda^*_{1,ed}(\tau_{ed}) = E_{ed}, \quad (23)$$

and the values of optimal τ_{ed} and E_{ed} can be obtained by solving the two following equations simultaneously:

$$\tau_{ed} = \frac{d}{du}\Lambda_{0,ed}(u_0) = \frac{d}{du}\Lambda_{1,ed}(u_1), \quad (24)$$

$$E_{ed} = \Lambda_{0,ed}(u_0) + (u-u_0)\frac{d}{du}\Lambda_{0,ed}(u_0)\Big|_{u=0} = \Lambda_{1,ed}(u_1) + (u-u_1)\frac{d}{du}\Lambda_{1,ed}(u_1)\Big|_{u=0}. \quad (25)$$

Proof: See [14].

Fig. 1 shows the optimal design with the two asymptotic CGFs. Since the Fenchel-Legendre transform is defined as $\Lambda^*_i(z) := \sup_{u\in\mathbb{R}}(zu - \Lambda_i(u))$, the error exponent is the y-intercept of the tangent line of $\Lambda_i(u)$ with slope τ_{ed}. Hence, the maximum of $\min\{\Lambda^*_{0,ed}(\tau_{ed}), \Lambda^*_{1,ed}(\tau_{ed})\}$ occurs when the two tangent lines of $\Lambda_{0,ed}$ and $\Lambda_{1,ed}$ coincide! Thus, for optimal performance some bias on the threshold should be applied. If we simply use the threshold $\tau_{opt} = 0$, then the error exponent is given by

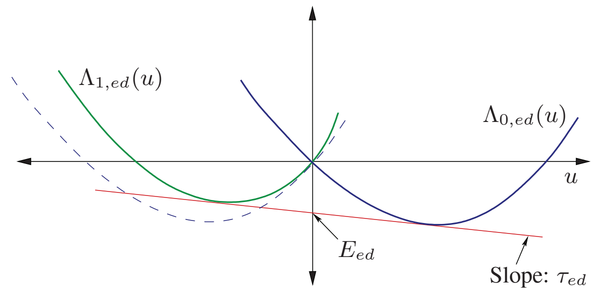

Fig. 1. The asymptotically optimal design for energy detection

$$E'_{ed} = -\min\{\Lambda^*_{0,ed}(0), \Lambda^*_{1,ed}(0)\} = -\Lambda^*_{1,ed}(0), = -\min_u \Lambda_{1,ed}(u) \le E_{ed}, \quad (26)$$

since $\Lambda_{1,ed}(u) \ge \Lambda_{0,ed}(u+1)$, which was shown in [14].

3.2. Example: Equi-correlated Signals

Based on the obtained error exponent for the energy detection, the Bahadur efficiency can be computed numerically. However, in certain cases including the equi-correlation signal model, closed-form expressions can be obtained. In this subsection, we provide the closed-form ARE for the energy detection in the case of the equi-correlated signal model which was widely used to capture the signal correlation in a simple way and to provides an insight into the energy detection of correlated signals [15].The equi-correlation signal model has the correlation coefficients, given by

$$\gamma_k = \mathbb{E}\{r[i]r[i-k]\} = \begin{cases} 1 & \text{if } k = 0 \\ \rho & \text{if } k \neq 0 \end{cases}, \quad (27)$$

where ρ is the correlation between any two samples. Here, the spectral density of the signal is given by

$$S_r(\omega) = 1 - \rho + \rho\delta(\omega). \quad (28)$$

Based on the result $\int_{-\pi}^{\pi}\log(a + b\delta(\omega))d\omega = 2\pi\log a$ for $a, b > 0$ [16], the error exponent for the optimal detection with zero threshold is given by

$$E_{opt} = \frac{1}{2}\log\left(\frac{\log\left(\frac{\sigma^2+\theta^2(1-\rho)}{\sigma^2}\right)}{\theta^2(1-\rho)/\sigma^2}\right) - \frac{1}{2}\frac{\log\left(\frac{\sigma^2+\theta^2(1-\rho)}{\sigma^2}\right)}{\theta^2(1-\rho)/\sigma^2} + \frac{1}{2}. \quad (29)$$

and the asymptotic CGFs for the energy detection are given by

$$\Lambda_{0,ed}(u) = -\frac{1}{2}\log\left(\frac{\sigma^2+\theta^2}{\sigma^2} - u\frac{\theta^2}{\sigma^2}\right) - \frac{u-1}{2}\log\left(\frac{\theta^2+\sigma^2}{\sigma^2}\right), \quad u < \frac{\sigma^2+\theta^2}{\theta^2}, \quad (30)$$

$$\Lambda_{1,ed}(u) = -\frac{1}{2}\log\left(1 - u\frac{\theta^2(\sigma^2+\theta^2(1-\rho))}{\sigma^2(\sigma^2+\theta^2)}\right) - \frac{u}{2}\log\left(\frac{\theta^2+\sigma^2}{\sigma^2}\right), \quad u \le 0. \quad (31)$$

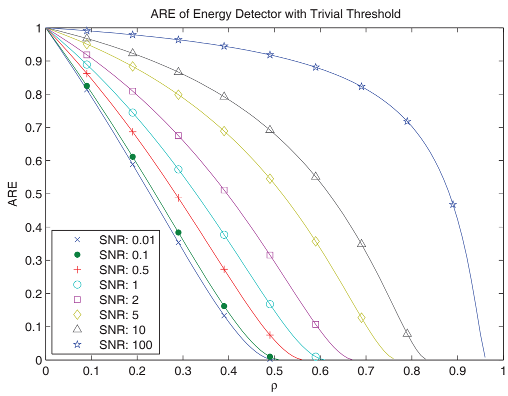

Fig. 2. Bahadur ARE of energy detector with trivial threshold

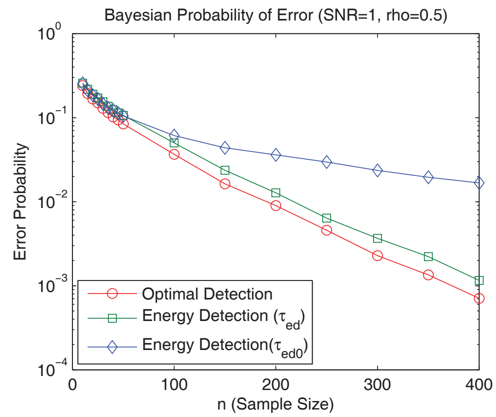

Fig. 3. Probability of error ($\theta^2/\sigma^2 = 1$, $\rho = 0.5$)

Now, by Theorem 3, we have

$$\tau_{ed} = \frac{1}{2}\frac{\sigma^2+\theta^2(1-\rho)}{(1-\rho)(\sigma^2+\theta^2)}\log\left(\frac{\sigma^2+\theta^2(1-\rho)}{\sigma^2}\right) - \frac{1}{2}\log\left(\frac{\sigma^2+\theta^2}{\sigma^2}\right),$$

$$E_{ed} = \frac{1}{2}\log\left(\frac{\log\left(\frac{\sigma^2+\theta^2(1-\rho)}{\sigma^2}\right)}{(1-\rho)\theta^2/\sigma^2}\right) - \frac{1}{2}\frac{\sigma^2}{(1-\rho)\theta^2}\log\left(\frac{\sigma^2+\theta^2(1-\rho)}{\sigma^2}\right) + \frac{1}{2}.$$

Surprisingly, the error exponent of the energy detector with the optimal threshold (not zero) is the same as that of the optimal detector itself. If the threshold $\tau_{opt} = 0$ for the optimal detector is used instead, the error exponent E'_{ed} is given by

$$E'_{ed} = \min \Lambda_{1,ed},$$
$$= \frac{1}{2}\log\left(\frac{\sigma^2(\sigma^2+\theta^2)\log\left(\frac{\sigma^2+\theta^2}{\sigma^2}\right)}{\theta^2(\sigma^2+\theta^2(1-\rho))}\right) - \frac{1}{2}\frac{\sigma^2(\sigma^2+\theta^2)\log\left(\frac{\sigma^2+\theta^2}{\sigma^2}\right)}{\theta^2(\sigma^2+\theta^2(1-\rho))} + \frac{1}{2}, \tag{32}$$

and the Bahadur efficiency can be obtained by using (29) and (32). The Bahadur ARE of the energy detector with threshold zero is plotted in Fig. 2. We observe that the ARE decreases as ρ increases. Interestingly, the ARE increases as SNR increases for the same value of ρ. (This property of ARE was proven in [5].)

To validate our asymptotic analysis based on the Bahadur efficiency, we provide some simulation result. We generated equi-correlated Gaussian signals with $\rho = 1/2$ and $\pi_0 = \pi_1 = 1/2$, and performed the different detection schemes: the optimal detection, the energy detection with the optimized threshold, and the energy detection with threshold zero. Fig. 3 shows the average detection error probability w.r.t. the sample size. Indeed, the energy detection with the optimized threshold has the same error slope as the optimal detector, whereas the energy detector with threshold zero has performance degradation.

4. CONCLUSION

In this paper, we have analyzed the asymptotic performance loss of energy detection compared with optimal detection, based on the the Bahadur ARE, which is the ratio of the error exponents of two detectors. Based on the Bahadur ARE, we have shown that the optimal threshold for optimal detection is not optimal for energy detection and that the optimal threshold for energy detection can be obtained by solving an integral equation. We have provided an example of the detection of equi-correlated Gaussian signals, and the numerical result validates our asymptotic analysis in the finite sample regime.

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