On the Performance of Randomly Directional Beamforming Between Line-of-Sight and Rich Scattering Channels

(Invited Paper)

Gilwon Lee*, Youngchul Sung* and Marios Kountouris†
* School of Electrical Engineering, KAIST, Daejeon 305-701, South Korea
† Mathematical and Algorithmic Sciences Lab, France Research Center, Huawei Technologies Co. Ltd.
Email: {gwlee@ and ysung@ee.}kaist.ac.kr, marios.kountouris@huawei.fr

***Abstract*—In this paper, the performance of random beamforming (RBF) which requires only partial channel state information (CSI) feedback is investigated for millimeter-wave (mm-wave) multiple-input multiple-output (MIMO) downlink systems under a new channel model that captures both the independent and identically-distributed (i.i.d.) Rayleigh fading MIMO channel model and the uniform random line-of-sight (UR-LoS) channel model and bridges the two channel models. Under the proposed channel model, we answer the basic question "how many users in the cell are required for RBF to achieve linear sum rate scaling with respect to (w.r.t.) the number of transmit antennas?"**

I. Introduction

Channel estimation and CSI feedback induce significant system overhead in multi-user (MU) MIMO downlink. To overcome the difficulty, the random beamforming (RBF) method was proposed in [1]. In this method, the base station (BS) picks a set of random orthogonal beams, selects a user that has the largest signal-to-interference-plus-noise ratio (SINR) for each random beam, and then simultaneously transmits data streams to the selected users with the associated set of random orthogonal beams. Thus, the RBF method eliminates the full CSI feedback but yields reasonable performance with partial CSI (scalar SINR not CSI) feedback only by exploiting the MU gain in the network. Due to such advantages, RBF and the associated MU gain have been investigated extensively over the last decade [1]–[5]. However, most of the performance analysis of RBF was performed under the rich scattering channel model that assumes independent Rayleigh fading for each element of the channel vector. Although this rich Rayleigh fading MIMO channel model is reasonable for small-scale MIMO systems at low frequency bands, the channel model is not suitable for large-scale MIMO systems in the mm-wave band, where the number of propagation paths is very few due to the quasi-optical nature of propagation in the mm-wave band [6], [7].

Recently, to overcome the difficulty in channel estimation in mm-wave large MIMO downlink [8]–[10], in [3] Lee *et al.* considered the RBF method in mm-wave MIMO downlink and analyzed its performance under the *uniform random line-of-sight (UR-LoS) channel model* which well captures the quasi-optical propagation characteristics in the mm-wave band [6], [7]. They showed very different behaviors of the RBF method* under the UR-LoS channel model from those under the conventional i.i.d. Rayleigh fading MIMO channel model. Explicitly, under the i.i.d. Rayleigh fading MIMO channel model, Sharif and Hassibi showed that the RBF sum rate $\mathcal{R}_{RBF}$ in a K-user multiple-input single-output (MISO) downlink system with M transmit antennas scales as [1]

$$\mathcal{R}_{RBF} \sim_K \begin{cases} M \log\log K, & \text{as } K \to \infty, \quad \text{for fixed } M, \\ cM, & \text{as } K \to \infty, \quad \text{for } M = O(\log K), \end{cases} \tag{1}$$

where $x \sim_K y$ indicates that $\lim_{K\to\infty} x/y = 1$, and c is a positive constant. Furthermore, they showed [1]

$$\lim_{K\to\infty} \frac{\mathcal{R}_{RBF}}{M} = 0, \tag{2}$$

provided that $\lim_{K\to\infty} \frac{\log K}{M} = 0$, i.e., $K = o(e^{c'M})$ for some constant c'. That is, if linear sum rate scaling w.r.t. the number of antennas is desired by the RBF scheme, a number of users exponentially increasing as a function of the number of transmit antennas is required in the cell. This result is quite pessimistic for the RBF scheme to be used in massive MIMO [5], [11]. However, Lee *et al.* recently showed that *sum rate scaling arbitrarily close to the linear behavior w.r.t. the number of transmit antennas is possible only with a linearly-increasing number of users w.r.t. the number of transmit antennas in the cell for the RBF scheme under the UR-LoS channel model* [3]. Their result sheds an optimistic prospect for the RBF scheme to be used in mm-wave massive MIMO. However, as the i.i.d. Rayleigh MIMO channel model is an extreme channel model, the UR-LoS channel model is another extreme channel model considering only one single LoS propagation path. The goal of the current paper is to investigate the performance of the RBF (or RDB) scheme under a more general channel model that can capture multiple propagation paths from the BS to each user in the cell.

Notation: Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^H$ and $\mathbf{A}^T$ indicate the conjugate transpose and

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*Under the UR-LoS channel model, orthogonal beams are constructed based on beam directions. So, they named the scheme as randomly-directional beamforming (RDB).

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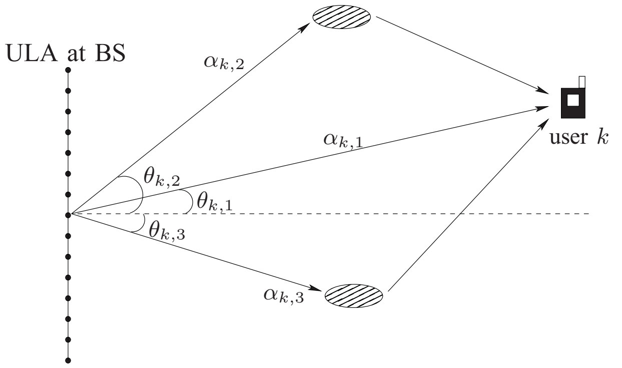

Fig. 1. The proposed uniform random multipath (UR-MP) channel model ($L = 3$): The uniform randomness is in the arrival-of-departure angle domain.

transpose of $\mathbf{A}$, respectively. $\mathbf{I}_M$ is the $M \times M$ identity matrix. $\mathbf{x} \sim \mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that random vector $\mathbf{x}$ is complex Gaussian distributed with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$, and $\theta \sim \text{Unif}(a, b)$ means that θ is uniformly distributed for $\theta \in [a, b)$. $\mathbb{E}[\cdot]$ denotes statistical expectation. $\iota = \sqrt{-1}$.

II. System Model

We consider a MU-MISO downlink system consisting of a BS equipped with a uniform linear array (ULA) of M transmit antennas and K single-antenna users. We consider the RDB scheme in this MU-MISO system which chooses M users among the K users in the cell and broadcasts independent data streams to the M selected users.

The received signal at user k is given by

$$y_k = \sqrt{\rho}\mathbf{g}_k^H \mathbf{w}_k x_k + \sqrt{\rho} \sum_{j \neq k} \mathbf{g}_k^H \mathbf{w}_j x_j + n_k, \tag{3}$$

where $\mathbf{g}_k, \mathbf{w}_k, x_k$, and $n_k \sim \mathcal{CN}(0, 1)$ are the channel vector, unit-norm beamforming vector, transmitted symbol, and additive noise for user k, respectively. Here, we assume $x_k \sim \mathcal{CN}(0, 1)$ and equal power allocation for each stream. Thus, $\rho = \frac{P_t}{M}$ is the average signal-to-noise ratio (SNR), where P_t is the total transmit power at the BS.

A. The Proposed Channel Model

Since our goal in this paper is to analyze the performance of RDB under a general channel model between the UR-LoS and i.i.d. Rayleigh fading MIMO channel models, we propose a new channel model that can capture both channel models and connect the two extreme channel models, by extending the UR-LoS channel model. In the proposed channel model, each channel vector $\mathbf{g}_k$ is given by

$$\mathbf{g}_k = \sqrt{\frac{M}{L}} \sum_{i=1}^{L} \alpha_{k,i} \mathbf{a}(\theta_{k,i}), \tag{4}$$

where L is the number of multiple paths; $\alpha_{k,i} \overset{i.i.d.}{\sim} \mathcal{CN}(0, 1)$ and $\theta_{k,i} \overset{i.i.d.}{\sim} \text{Unif}[-1, 1]$ are the path gain and normalized angle-of-departure (AoD)† of the i-th path for channel vector k, respectively; and $\mathbf{a}(\theta)$ is the normalized array steering vector given by

$$\mathbf{a}(\theta) = \frac{1}{\sqrt{M}}[1, e^{-\iota\pi\theta}, \cdots, e^{\iota\pi(M-1)\theta}]^T. \tag{5}$$

Here, the normalized AoD $\theta \in [-1, 1]$ is related to the physical AoD $\phi \in [-\pi/2, \pi/2]$ as

$$\theta = \frac{2d \sin(\phi)}{\lambda},$$

where d and λ are the distance between two adjacent antenna elements and the carrier wavelength, respectively. We assume that $\frac{d}{\lambda} = \frac{1}{2}$ in this paper. Note that the channel vector $\mathbf{g}_k$ in the proposed channel model is the sum of L uniform random multi-paths (UR-MP) with complex Gaussian gains. Thus, we refer to this channel model as the *uniform random multi-path (UR-MP) channel model*. Note that the UR-MP channel model reduces to the widely-considered conventional channel models by controlling L.

Case i) The case of $L = 1$: In this case, the proposed channel model is expressed as

$$\mathbf{g}_k = \sqrt{M} \alpha_{k,1} \mathbf{a}(\theta_{k,1}), \tag{6}$$

which is the UR-LoS channel model considered in [3], [7].

Case ii) The case of $L \to \infty$ for fixed M: In this case, by the law of large numbers (LLN), we have

$$\mathbf{g}_k \to \mathbf{h}_k, \quad \text{as} \quad L \to \infty \tag{7}$$

where $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. That is, the proposed channel model converges to the i.i.d. Rayleigh fading MIMO channel model when L tends to infinity for fixed M.

In order to consider the massive MIMO asymptote as in mm-wave MIMO systems with highly-directional beamforming with large M, we need to consider the case that the number M of transmit antennas tends to infinity. In case of $M \to \infty$, the condition that $L \to \infty$ is not sufficient for the UR-MP channel model to converge to the i.i.d. Rayleigh fading channel model. Thus, we consider the following channel model:

Definition 1 (The UR-MP channel model with parameter β): The UR-MP channel model with parameter β is defined as the channel model (4) with $L = M^\beta$ for some $\beta \in [0, \infty)$.

In the UR-MP channel model with parameter β, the parameter β determines the richness in the number of multi-paths w.r.t. the number of transmit antennas. The following theorem provides the behavior of the channel model depending on β.

Theorem 1: The UR-MP channel model with parameter β converges to the i.i.d. Rayleigh fading MIMO channel model, as $M, L \to \infty$, when $\beta > 1$, i.e.,

$$\mathbf{g}_k \to \mathbf{h}_k, \tag{8}$$

†Since we consider the MISO case, AoD matters. We assume that there exists a scatter or reflector at each AoD included in the model (4) to generate a propagation path from the BS and user k at that AoD.

where $\mathbf{h}_k \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_M)$. On the other hand, the UR-MP channel model with parameter β does not converges to the i.i.d. Rayleigh fading MIMO channel model, as $M, L \to \infty$, when $0 \leq \beta < 1$. In this case, the covariance matrix of the channel vector is not of full rank.

Proof: Proof is omitted due to space limitation. ■

Note that the UR-MP channel model is general to include the UR-LoS channel model by setting $\beta = 0$ and the i.i.d. Rayleigh fading channel model by setting $\beta > 1$. When $\beta \in (0, 1)$, the channel model lies somewhere in-between the two extreme channel models. The parameter β monotonically represents the richness in the number of multi-paths in the propagation between the BS and the considered user.

III. Performance Analysis

In this section, we briefly explain the RDB scheme in [3] which is a special case of RBF of [1]. Then, we analyze the performance of RDB under the UR-MP channel model with parameter β for $0 < \beta < 1$. Specifically, we identify how many users K are required to achieve linear scaling of the RDB throughput w.r.t. the number M of antennas under the UR-MP channel model.

In RBF, the BS chooses a set of random orthonormal beam vectors $\{\mathbf{w}_b\}_{b=1}^M$ that forms an orthonormal basis of $\mathbb{C}^M$, selects a user that has the maximum SINR for each random beam, and transmit M independent data streams to the M selected users. Then, the expected sum rate of the RBF method is given by

$$\mathcal{R}_{sum} = \sum_{b=1}^{M} \mathbb{E}\left[\log\left(1 + \text{SINR}_{\kappa_b, b}\right)\right] = \sum_{b=1}^{M} \mathcal{R}_{\kappa_b}, \quad (9)$$

where

$$\text{SINR}_{k,b} = \frac{\rho |\mathbf{g}_k^H \mathbf{w}_b|^2}{1 + \rho \sum_{b' \neq b} |\mathbf{g}_k^H \mathbf{w}_{b'}|^2} \quad (10)$$

and $\kappa_b = \arg\max_{1 \leq k \leq K} \text{SINR}_{k,b}$. In the case of RDB scheme, the M transmit beams are constructed with different beam directions. That is, the BS chooses a special orthonormal basis $\{\mathbf{w}_b\}_{b=1}^M$ as

$$\mathbf{w}_b = \mathbf{a}(\vartheta_b) = \mathbf{a}\left(\vartheta + \frac{2(b-1)}{M}\right), \quad \text{for} \quad b = 1, \cdots, M, \quad (11)$$

where $\mathbf{a}(\cdot)$ is defined in (5), and $\vartheta \sim \text{Unif}[-1, 1]$ or equivalently $\vartheta \sim \text{Unif}[-1, 1 + \frac{2}{M}]$ is a random offset value. That is, the RDB uses M beams equi-spaced in the normalized angle domain with a uniform random offset. Note that in the RDB scheme, the inter-beam interference results when the user is not located at the exact boresight angle but located somewhere in-between equi-spaced boresight beam angles.

Before we investigate the RDB performance for the UR-MP channel model with parameter β, we consider the UR-MP channel model with finite and fixed L regardless of M.

Theorem 2: For $K = M^q$ with $q = 1 + \epsilon$ and arbitrary $\epsilon > 0$, under the UR-MP model with finite and fixed L, an asymptotic lower bound on the per-user rate $\mathcal{R}_{\kappa_b}$ for fixed total transmit power $P_t = 1$ is given by

$$\mathcal{R}_{\kappa_b} = \mathbb{E}\left[\log\left(1 + \frac{M^{-1}|\mathbf{g}_{\kappa_b}^H \mathbf{a}(\vartheta_b)|^2}{1 + M^{-1}\sum_{b' \neq b} |\mathbf{g}_{\kappa_b}^H \mathbf{a}(\vartheta_{b'})|^2}\right)\right] \gtrsim c > 0, \quad (12)$$

where $x \gtrsim y$ indicates $\lim_{M \to \infty} \frac{x}{y} \geq 1$, and c is a positive constant value.

Proof: Let A be the event that there exists a user k' such that $\theta_{k',1} \in [\vartheta_b - \frac{1}{M}, \vartheta_b + \frac{1}{M})$, $|\alpha_{k',1}| \geq L$, and $|\alpha_{k',i}| \leq 1$, $\theta_{k',i} \notin [\vartheta_b - \frac{1}{M}, \vartheta_b + \frac{1}{M})$ for $i \neq 1$. Note that we have for any k, i

$$\Pr\left\{\theta_{k,i} \in \left[\vartheta_b - \frac{1}{M}, \vartheta_b + \frac{1}{M}\right)\right\} = \frac{1}{M} \quad (13)$$

$$\Pr\{|\alpha_{k,i}| \geq L\} = e^{-L^2}. \quad (14)$$

Using the above, we compute the asymptotic probability of the event A as follows:

$$\begin{aligned}
\Pr\{A\} &= 1 - \Pr\{A^c\} \\
&= 1 - \left(1 - \frac{1}{M}e^{-L^2}\left((1 - e^{-1})(1 - \frac{1}{M})\right)^{L-1}\right)^K \\
&= 1 - \left(1 - O\left(\frac{1}{M}\right)\right)^{M^q} \\
&= 1 - e^{M^q \log\left(1 - O\left(\frac{1}{M}\right)\right)} \\
&\overset{(a)}{=} 1 - e^{-M^{q-1} + O\left(\frac{1}{M^{2-q}}\right)} \to 1
\end{aligned} \quad (15)$$

as $M \to \infty$, where (a) follows from $\log(1 - x) = -x + O(x^2)$ for small x. Now, from the fact that $\mathbb{E}[f(X)] \geq p(A)\mathbb{E}[f(X|A)]$ for a non-negative function $f(X)$, $\mathcal{R}_{\kappa_b}$ can be lower bounded by

$$\mathcal{R}_{\kappa_b} \geq \Pr\{A\}\mathbb{E}\left[\log\left(1 + \frac{\frac{1}{M}|\mathbf{g}_{\kappa_b}^H \mathbf{a}(\vartheta_b)|^2}{1 + \frac{1}{M}\sum_{b' \neq b} |\mathbf{g}_{\kappa_b}^H \mathbf{a}(\vartheta_{b'})|^2}\right) \middle| A\right] \quad (16)$$

The second term in the right-hand side (RHS) of (16) can further be bounded as

$$\begin{aligned}
&\mathbb{E}\left[\log\left(1 + \frac{\frac{1}{M}|\mathbf{g}_{\kappa_b}^H \mathbf{a}(\vartheta_b)|^2}{1 + \frac{1}{M}\sum_{b' \neq b} |\mathbf{g}_{\kappa_b}^H \mathbf{a}(\vartheta_{b'})|^2}\right) \middle| A\right] \\
&\overset{(a)}{\geq} \mathbb{E}\left[\log\left(1 + \frac{\frac{1}{L}|\sum_{i=1}^L \alpha_{k',i}^* \mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)|^2}{1 + \frac{1}{L}\sum_{b' \neq b}\left|\sum_{i=1}^L \alpha_{k',i}^* \mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})\right|^2}\right) \middle| A\right] \\
&\overset{(b)}{\geq} \mathbb{E}\left[\log\left(1 + \frac{\frac{1}{L}(|\alpha_{k',1}| - (L-1))^2 \frac{4}{\pi^2}}{1 + \frac{1}{L}(|\alpha_{k',1}|^2 + (L-1))\frac{L\pi^2}{3}}\right) \middle| A\right] \\
&\overset{(c)}{\geq} \log\left(1 + \frac{\frac{4}{L\pi^2}}{1 + \frac{(L^2+L-1)\pi^2}{3}}\right) =: c > 0,
\end{aligned} \quad (17)$$

where (a) follows from the fact that we compute the rate of user k' instead of user κ_b; (b) holds by the two facts

$$|\sum_{i=1}^{L} \alpha_{k',i} \mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)|^2 \geq (|\alpha_{k',1}| - (L-1))^2 \frac{4}{\pi^2} \quad (18)$$

and

$$\sum_{b'\neq b}\left|\sum_{i=1}^{L}\alpha_{k',i}\mathbf{a}(\theta_{k',i})^H\mathbf{a}(\vartheta_{b'})\right|^2 \leq (|\alpha_{k',1}|^2+(L-1))\frac{L\pi^2}{3}. \quad (19)$$

(See Appendix.); and (c) follows from the fact that the SINR term is minimized when $|\alpha_{k',1}| = L$ (due to $|\alpha_{k',1}| \geq L$). Therefore, by using (16) with the fact that $\Pr\{A\} \to 1$ and (17), we have $\mathcal{R}_{\kappa_b} \gtrsim c > 0$. ■

Theorem 2 states that under the UR-MP channel model with finite L, linear sum rate scaling w.r.t. the number of transmit antennas by the RDB scheme is achievable when the number of users in the cell increases arbitrarily close to linearly w.r.t. the number of transmit antennas. This result is not much different from that under the UR-LoS channel model provided in [3]. Thus, the RDB scheme is promising for mm-wave massive MIMO systems even for multiple propagation paths if the number of multi-paths is finite.

Now, consider the RDB performance under the UR-MP channel model with parameter β. In this case, we have the following theorem.

Theorem 3: Under the UR-MP channel model with parameter β with $L = M^\beta$ and $\beta \in (0,1)$, for $K = ML^q$ and $q \geq 3L$, an asymptotic lower bound on the per-user rate $\mathcal{R}_{\kappa_b}$ for fixed total transmit power $P_t = 1$ is given by

$$\begin{aligned}\mathcal{R}_{\kappa_b} &= \mathbb{E}\left[\log\left(1+\frac{M^{-1}|\mathbf{g}_{\kappa_b}^H\mathbf{a}(\vartheta_b)|^2}{1+M^{-1}\sum_{b'\neq b}|\mathbf{g}_{\kappa_b}^H\mathbf{a}(\vartheta_{b'})|^2}\right)\right]\\ &\gtrsim c^\star > 0 \end{aligned} \quad (20)$$

and $c^\star$ is a positive constant value.

Proof: Proof is similar to the proof of Theorem 2. Without loss of generality, we assume $\vartheta_b = 0$. Let $A^\star$ be the event that there exists a user $k^\star$ such that $|\theta_{k^\star,1}| \in [0, \frac{1}{M})$, $|\alpha_{k^\star,1}| \in [\sqrt{L}, \sqrt{2L}]$, and for $i \neq 1$, $|\theta_{k^\star,i}| \in [\frac{1}{M}+\frac{(i-1)}{L}, \frac{1}{M}+\frac{i}{L})$ and $|\alpha_{k^\star,i}| \leq 1$. Now, we compute the asymptotic probability of the event $A^\star$ as follows:

$$\begin{aligned}\Pr\{A^\star\} &= 1-\left(1-\frac{1}{M}(e^{-L}-e^{-2L})\left((1-e^{-1})\frac{1}{L}\right)^{L-1}\right)^K\\ &\geq 1-\left(1-\frac{1}{Me^{2L}L^L}\right)^K\\ &= 1-e^{e^{q\log L+\log M}\log\left(1-\frac{1}{e^{2L+L\log L+\log M}}\right)}\\ &= 1-e^{-e^{q\log L-2L-L\log L}+o(1)} \to 1 \end{aligned} \quad (21)$$

as $M \to \infty$. Here we used $\log(1-x) = -x + O(x^2)$ for small x and the condition $q \geq 3L$. To apply similar techniques used in (16) and (17), we should find a lower bound of $|\sum_{i=1}^{L}\alpha_{k^\star,i}\mathbf{a}(\theta_{k^\star,i})^H\mathbf{a}(\vartheta_b)|^2$ and an upper bound of $\sum_{b'\neq b}\left|\sum_{i=1}^{L}\alpha_{k^\star,i}\mathbf{a}(\theta_{k^\star,i})^H\mathbf{a}(\vartheta_{b'})\right|^2$. By Lemma 2 in [3], for all b,

$$\begin{aligned}|\sum_{i\neq 1}\alpha_{k^\star,i}\mathbf{a}(\theta_{k^\star,i})^H\mathbf{a}(\vartheta_b)| &\leq \sum_{i\neq 1}|\alpha_{k^\star,i}||\mathbf{a}(\theta_{k^\star,i})^H\mathbf{a}(\vartheta_b)|\\ &\leq \sum_{i\neq 1}\frac{1}{M(i-1)/L}\\ &\leq \frac{\beta\log M}{M^{1-\beta}}. \end{aligned} \quad (22)$$

Therefore, we have

$$\begin{aligned}|\sum_{i=1}^{L}\alpha_{k^\star,i}\mathbf{a}(\theta_{k^\star,i})^H\mathbf{a}(\vartheta_b)|^2 &\geq \left(\frac{2}{\pi}M^{\frac{1}{2}\beta}-\frac{\beta\log M}{M^{1-\beta}}\right)^2\\ &\to \frac{4}{\pi^2}M^\beta, \end{aligned} \quad (23)$$

as $M \to \infty$. Also, we can re-arrange the indices of $\{\vartheta_{b'}\}_{b'\neq b}$ in the order of closeness to ϑ_b with the new indices $\{j\}$. Then we have by Lemma 2 in [3],

$$|\mathbf{a}(\theta_{k^\star,1})^H\mathbf{a}(\vartheta_{2j})|, |\mathbf{a}(\theta_{k^\star,1})^H\mathbf{a}(\vartheta_{2j-1})| \leq \frac{1}{2j} \quad (24)$$

Therefore, by (22), (24) and $|\alpha_{k^\star,1}| \leq \sqrt{2L}$, we have

$$\begin{aligned}\sum_{j\neq b}\left|\sum_{i=1}^{L}\alpha_{k^\star,i}\mathbf{a}(\theta_{k^\star,i})^H\mathbf{a}(\vartheta_j)\right|^2 &\leq \sum_{j\neq b}\left|M^{\frac{1}{2}\beta}\cdot\frac{1}{\sqrt{2}j}+\frac{\beta\log M}{M^{1-\beta}}\right|^2\\ &\leq \sum_{j=1}^{M/2}2\left|M^{\frac{1}{2}\beta}\cdot\frac{1}{j}\right|^2+4\left|\frac{\beta\log M}{M^{1-\beta}}\right|^2\\ &\sim \frac{\pi^2}{3}M^\beta \end{aligned} \quad (25)$$

Therefore, by using (21), (23), and (25), we have

$$\mathcal{R}_{\kappa_b} \gtrsim \log\left(1+\frac{\frac{4}{\pi^2}}{1+\frac{\pi^2}{3}}\right) =: c^\star > 0 \quad (26)$$

■

Theorem 3 states that linear sum rate scaling by the RDB scheme is achievable under the UR-MP channel model with parameter $\beta \in (0,1)$ when $K = ML^q$ and $q \geq 3L$. Consider the condition of $K = ML^q$ and $q \geq 3L$. By applying $L = M^\beta$, we have

$$K = ML^q = M(M^\beta)^{3M^\beta}. \quad (27)$$

Taking logarithm on both sides of (27), we have

$$\log K = (1+3\beta M^\beta)\log M, \quad 0<\beta<1. \quad (28)$$

When $\beta = 0$, the sufficient condition reduces to $\log K = \log M$, which coincides with the previous result for the UR-LoS channel model. On the other hand, when $0<\beta<1$, we have

$$\Theta(\log K) = M^\beta \log M. \quad (29)$$

Notice the difference between the two respective sufficient conditions (1) (i.e., $M = O(\log K)$) and (29) for linear sum rate scaling for the RBF or RDB scheme under the i.i.d. Rayleigh fading and UR-MP with parameter β channel models. When $\beta \uparrow 1$, i.e., the UR-MP channel model with parameter β converges to the i.i.d. Rayleigh fading channel model by Theorem 1, the obtained result is a bit loose than the result of $M = O(\log K)$ in [1]. However, when $\beta < 1$, we have $\frac{M^\beta \log M}{M} \to 0$ as $M \to \infty$.

IV. Conclusion

In this paper, we have examined the performance of the RDB scheme under the newly proposed UR-MP channel model that captures the UR-LoS and Rayleigh fading channel models and connects the two extreme models. We have shown that linear scaling of the RDB throughput w.r.t. the number M of transmit antennas can be achieved under the UR-MP model with a finite and fixed number L of multi-paths, if $K = M^{1+\epsilon}$ for any $\epsilon > 0$, as $M \to \infty$, and that linear scaling of the RDB throughput w.r.t. M can be achieved under the UR-MP model with $L = M^\beta$, if $\Theta(\log K) = M^\beta \log M$, as $M \to \infty$, for $0 < \beta < 1$. Thus, in sparse multi-path channels the number of required users for RDB to achieve linear sum rate scaling w.r.t. M is far less than that required in i.i.d. Rayleigh fading MIMO channels.

Appendix

Proof of eq. (18)*:* Consider the value

$$\left|\sum_{i=1}^{L} \alpha_{k',i}\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)\right|$$
$$= \left|\alpha_{k',1}\mathbf{a}(\theta_{k',1})^H \mathbf{a}(\vartheta_b) + \sum_{i\neq 1} \alpha_{k',i}\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)\right|. \quad (30)$$

Here, we note that $|\alpha_{k',1}| \geq L$ and $|\mathbf{a}(\theta_{k',1})^H \mathbf{a}(\vartheta_b)| \geq \frac{2}{\pi}$ due to $|\theta_{k',1} - \vartheta_b| \leq \frac{1}{M}$ [3]. For $i \neq 1$, we have $|\alpha_{k',i}| \leq 1$, and $|\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)| \leq |\mathbf{a}(\theta_{k',1})^H \mathbf{a}(\vartheta_b)|$ because of $|\theta_{k',i} - \vartheta_b| \geq \frac{1}{M}$ [3]. Therefore, the magnitude of the first term in (30) is greater than the magnitude of the second term in (30). So (30) is minimized when their phases are opposite. Hence we have the lower bound of (30), given by

$$\begin{aligned}
&\left|\sum_{i=1}^{L} \alpha_{k',i}\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)\right| \\
&\geq \left|\alpha_{k',1}\mathbf{a}(\theta_{k',1})^H \mathbf{a}(\vartheta_b)\right| - \left|\sum_{i\neq 1} \alpha_{k',i}\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)\right| \\
&\geq |\alpha_{k',1}||\mathbf{a}(\theta_{k',1})^H \mathbf{a}(\vartheta_b)| - \sum_{i\neq 1} |\alpha_{k',i}| \left|\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_b)\right| \\
&\geq (|\alpha_{k',1}| - (L-1)) \left|\mathbf{a}(\theta_{k',1})^H \mathbf{a}(\vartheta_b)\right| \\
&\geq (|\alpha_{k',1}| - (L-1)) \frac{2}{\pi}
\end{aligned}$$

which concludes the proof. ■

Proof of eq. (19)*:* We have

$$\begin{aligned}
&\sum_{b'\neq b} \left|\sum_{i=1}^{L} \alpha_{k',i}\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})\right|^2 \leq \sum_{b'=1}^{M} \left|\sum_{i=1}^{L} \alpha_{k',i}\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})\right|^2 \\
&\leq \sum_{b'=1}^{M} \left(\sum_{i=1}^{L} |\alpha_{k',i}| \cdot \left|\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})\right|\right)^2 \\
&\overset{(a)}{\leq} \sum_{b'=1}^{M} L \sum_{i=1}^{L} \left(|\alpha_{k',i}| \cdot \left|\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})\right|\right)^2 \\
&\overset{(b)}{\leq} L \sum_{i=1}^{L} |\alpha_{k',i}|^2 \sum_{b'=1}^{M} \left|\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})\right|^2 \\
&\overset{(c)}{\leq} L \left(\sum_{i=1}^{L} |\alpha_{k',i}|^2\right) \frac{\pi^2}{3} \\
&\leq \left(|\alpha_{k',1}|^2 + (L-1)\right) \frac{L\pi^2}{3}
\end{aligned}$$

where (a) follows from Jensen's inequality for the convex function $f(x) = x^2$; (b) holds by interchanging the order of summation; (c) follows from the fact that

$$\begin{aligned}
\sum_{b'=1}^{M} |\mathbf{a}(\theta_{k',i})^H \mathbf{a}(\vartheta_{b'})|^2 &\leq \sum_{b'=1}^{M} \frac{1}{M^2|\theta_{k',i} - \vartheta_{b'}|^2} \\
&\leq \sum_{j=1}^{M/2} \frac{2}{j^2} \leq \frac{\pi^2}{3}
\end{aligned}$$

which is obtained by Lemma 2 in [3], where the second equality is obtained by re-arranging the indices of $\{\vartheta_{b'}\}$ and applying a similar technique used in eq. (66) of [3]. ■

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