

Precoder Design for Blind Channel Estimation in Multiple OFDM Systems

Song Noh* and Youngchul Sung†

Dept. of Electrical Engineering, KAIST, Daejeon 305-701 South Korea

* E-mail: nosong@kaist.ac.kr Tel: +82-42-350-5484

† E-mail: ysung@ee.kaist.ac.kr Tel: +82-42-350-3484

Abstract—The problem of blind channel estimation for multiple single-antenna orthogonal frequency division multiplexing (OFDM) systems coexisting in the same band is considered. Based on the previous work on linear precoding for blind channel estimation for OFDM systems, a new precoder design based on zero-hole matrices is proposed and optimal design is derived. Numerical results show the validity of our design. The proposed method can be used to design precoders to separate and estimate the channels from multiple basestations to a terminal station in cell edge even in case of colliding pilot tones in full frequency reuse.

I. INTRODUCTION

Due to its bandwidth efficiency blind channel estimation has gained substantial research interest for orthogonal frequency division multiplexing (OFDM) systems over recent years. This is especially the case for single or multiple user multi-input multi-output (MIMO) OFDM systems to which efficient subspace methods based on second order statistics [1] can easily be applied and several methods have been developed in time or frequency domain, e.g. [2],[3]. For multi-user single antenna systems, however, not much results is available for blind channel estimation since the direct application of subspace techniques is not straightforward. Recently, Liao et al. developed an effective blind channel estimation technique for single antenna OFDM systems based on linear precoding at transmitter [4]¹. However, the optimal solution to precoder design was not obtained in their work. Based on their result, we consider the optimal precoder design problem for the proposed blind estimation technique under the situation of multiple OFDM systems with co-channel interference in this paper. This case is especially useful to current and future wireless OFDM systems in which the frequency reuse factor is one and different users (from different cells) use the same subcarriers resulting co-channel interference. Our approach to multi-user blind channel estimation in single antenna OFDM systems is based on *non-overlapping precoding*. Under reasonable assumption, we have derived the optimal solution for non-overlapping precoder design for blind estimation. The proposed method provides an effective separation among users

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0021269)

¹Even if the authors developed the technique for two-way relay channels, the application to multiple OFDM systems is straightforward.

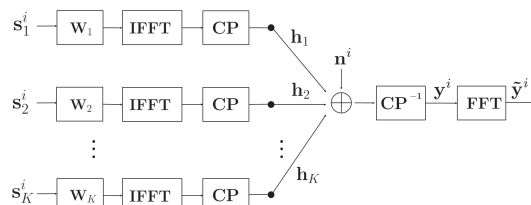


Fig. 1. System model

for blind channel estimation even with the frequency reuse factor of one.

The remaining of the paper is as follows. In Section II, the data model and previous work are presented. In Section III, we provide an improved version of the blind estimation technique, and the optimal precoder design is presented in Section IV. Some numerical results are provided in Section V, followed by conclusion in Section VI.

II. DATA MODEL

We consider a single antenna multiuser OFDM network in which all K users transmit their signal simultaneously and all users' signal are received at the receiver, as shown in Fig. 1. The i -th OFDM symbol of the k -th user is represented as a vector $\mathbf{s}_k^i = [s_k^i(0), \dots, s_k^i(N-1)]^T$, where N is the number of subcarriers in the system. At the transmitter, each user's data vector is processed with a linear precoder \mathbf{W}_k and then OFDM modulated, i.e., processed by inverse Discrete Fourier Transform (IDFT) and attached by cyclic prefix which is longer than the channel length. The transmitted signal from the k -th user passes through a finite impulse response (FIR) channel $\mathbf{h}_k = [h_k(0), \dots, h_k(L)]^T$, $k \in \{1, \dots, K\}$ with length $L+1$ to the receiver. Here, we assume that the channels are time-invariant over a block of N_s successive OFDM symbols. The received OFDM symbol at the receiver, after removing of the cyclic prefix portion, is given by

$$\mathbf{y}_i = \sum_{k=1}^K \mathbf{H}_k \mathbf{F}^H \mathbf{W}_k \mathbf{s}_k^i + \mathbf{n}^i, \quad (1)$$

where \mathbf{F}^H is the normalized IDFT matrix, \mathbf{n}^i a zero-mean spatio-temporally uncorrelated noise with covariance matrix $\sigma_n^2 \mathbf{I}$, and \mathbf{H}_k is the circulant channel matrix with the first row $[\mathbf{h}_k^T, 0, \dots, 0]$. The DFT of \mathbf{y}_i yields

$$\tilde{\mathbf{y}}_i = \sum_{k=1}^K \tilde{\mathbf{H}}_k \mathbf{W}_k \mathbf{s}_k^i + \tilde{\mathbf{n}}^i, \quad (2)$$

where $\tilde{\mathbf{H}}_k$ is the diagonal matrix of which diagonal elements are given by

$$\tilde{\mathbf{h}}_k = \tilde{\mathbf{F}}\mathbf{h}_k. \quad (3)$$

and $\tilde{\mathbf{F}}$ is the $N \times (L+1)$ matrix consisting of the first $L+1$ columns of the $N \times N$ normalized DFT matrix \mathbf{F} .

A. Previous Work [4]

Here, we briefly review the previous work on the blind channel estimation based on linear precoding by Liao et al. [4] which is relevant to our development in the next sections. The method is based on precoding and subspace technique with second-order statistics. Note that the covariance matrix of the received signal is given by

$$\mathbf{R} \triangleq \mathbb{E}\{\tilde{\mathbf{y}}_i \tilde{\mathbf{y}}_i^H\}, \quad (4)$$

$$= \tilde{\mathbf{H}}_1 \mathbf{Q}_1 \tilde{\mathbf{H}}_1^H + \dots + \tilde{\mathbf{H}}_K \mathbf{Q}_K \tilde{\mathbf{H}}_K^H + \sigma_n^2 \mathbf{I} \quad (5)$$

$$= \underbrace{\left(\sum_{k=1}^K \tilde{\mathbf{h}}_k \tilde{\mathbf{h}}_k^H \right)}_{=\tilde{\mathbf{J}}} \odot \mathbf{Q} + \sigma_n^2 \mathbf{I}, \quad (6)$$

where $\mathbf{Q}_k \triangleq \mathbf{W}_k \mathbf{W}_k^H$ and \odot denotes the Hadamard product. Here, it was assumed that all \mathbf{Q}_k matrices were equal for the third equality. Thus, the elements of \mathbf{R} are given by

$$[\mathbf{R}]_{i,i} = \sum_{k=1}^K |\tilde{h}_k(i)|^2 [\mathbf{Q}]_{i,i} + \sigma_n^2, \quad (7)$$

$$[\mathbf{R}]_{i,j} = \sum_{k=1}^K \tilde{h}_k(i) \tilde{h}_k^*(j) [\mathbf{Q}]_{i,j}, \quad i \neq j. \quad (8)$$

The off-diagonal elements of $\tilde{\mathbf{J}}$ can be obtained by

$$[\tilde{\mathbf{J}}]_{i,j} = \sum_{k=1}^K \tilde{h}_k(i) \tilde{h}_k^*(j) = [\mathbf{R}]_{i,j} / [\mathbf{Q}]_{i,j}. \quad (9)$$

Note here that \mathbf{R} can be obtained from data and \mathbf{Q} is under our design. Now, define

$$\mathbf{v}_j \triangleq \begin{bmatrix} [\mathbf{R}]_{1,j}, \dots, [\mathbf{R}]_{j-1,j}, [\mathbf{R}]_{j+1,j}, \dots, [\mathbf{R}]_{N,j} \\ [\mathbf{Q}]_{1,j}, \dots, [\mathbf{Q}]_{j-1,j}, [\mathbf{Q}]_{j+1,j}, \dots, [\mathbf{Q}]_{N,j} \end{bmatrix}^T \quad (10)$$

$$\mathbf{F}_j \triangleq \begin{bmatrix} \tilde{\mathbf{F}}(1:j-1) \\ \tilde{\mathbf{F}}(j+1:N) \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad (11)$$

Applying (3), we have

$$\mathbf{J} \triangleq \sum_{k=1}^K \mathbf{h}_k \mathbf{h}_k^H, \quad (12)$$

$$= \left[\mathbf{F}_1^\dagger \mathbf{v}_1, \dots, \mathbf{F}_N^\dagger \mathbf{v}_N \right] \tilde{\mathbf{F}}, \quad (13)$$

since $\left[\mathbf{F}_1^\dagger \mathbf{v}_1, \dots, \mathbf{F}_N^\dagger \mathbf{v}_N \right] = \sum_k \mathbf{h}_k \tilde{\mathbf{h}}_k^H$. Here, $(\cdot)^\dagger$ denotes the pseudo-inverse. Eq. (13) provides a way to identify the channel blindly. Once \mathbf{J} is constructed from \mathbf{v}_j , $j = 0, 1, \dots, N-1$, obtained from the data covariance matrix \mathbf{R} , we can obtain the subspace of $\{\mathbf{h}_k\}$ by low rank approximation via singular value decomposition (SVD) of \mathbf{J} [5]. That is, let

$$\mathbf{J} = \mathbf{U} \mathbf{\Sigma} \mathbf{U}^H. \quad (14)$$

Then, an estimate for the channel is given by

$$[\mathbf{h}_1, \dots, \mathbf{h}_K] = \mathbf{U}(:, 1:K) \mathbf{\Sigma}_K \mathbf{V}^H, \quad (15)$$

where $\mathbf{\Sigma}_K$ is the K dominant singular values in $\mathbf{\Sigma}$ and \mathbf{V} is a ambiguity factor with $\mathbf{V}^H \mathbf{V} = \mathbf{I}$. The remaining ambiguity \mathbf{V} can be resolved by embedded pilot symbols. Note here that without precoding, i.e., $\mathbf{Q} = \mathbf{I}$ such identification is not available.

III. BLIND ESTIMATION WITH SUBCARRIER SELECTION

To develop a precoder design strategy in the next section, we need to modify the discussed algorithm more efficiently. The modification is based on that all \mathbf{v}_j , $j = 1, \dots, N$ are not required to construct \mathbf{J} but $T (\geq L+1)$ subcarriers are enough. We first analyze the single-user case, and then the procedure will be generalized to the multi-user case.

A. Single User Case ($K = 1$)

Here, we show that how the submatrix of the covariance matrix can be used to estimate the channel parameter \mathbf{h} . When $K = 1$, eq. (6) reduces to

$$\mathbf{R} = \underbrace{\tilde{\mathbf{h}} \tilde{\mathbf{h}}^H}_{\tilde{\mathbf{J}}} \odot \mathbf{Q} + \sigma_n^2 \mathbf{I}, \quad (16)$$

and we have

$$[\tilde{\mathbf{J}}]_{i,j} = [\mathbf{R}]_{i,j} / [\mathbf{Q}]_{i,j} = \tilde{h}(i) \tilde{h}^*(j), \quad j \neq i. \quad (17)$$

Note first that $T (\geq L+1)$ subcarriers are sufficient to estimate the channel vector \mathbf{h} [6]. Suppose first that T rows of $\tilde{\mathbf{J}}$ are selected and $\mathcal{I} \subset \{0, 1, \dots, N-1\}$ is the index set of selected rows. Define $\tilde{\mathbf{F}}(\mathcal{I}, :)$ be the matrix composed of the rows \mathcal{I} of $\tilde{\mathbf{F}}$. Here, we assume that T rows with equi-spacing are selected². Then, \mathcal{I} is given by

$$\mathcal{I} = \{i_1, \dots, i_T\}, \quad (18)$$

where $i_l = (l-1) \lfloor \frac{N}{T} \rfloor + c$ and $c \in \{2, \dots, \lfloor \frac{N}{T} \rfloor\}$. Also, let \mathcal{J} be the index of selected columns of \mathcal{J} and

$$\mathcal{J} = \{j_1, \dots, j_T\}, \quad (19)$$

where $j_l = (l-1) \lfloor \frac{N}{T} \rfloor + d$, $d \in \{1, \dots, \lfloor \frac{N}{T} \rfloor\}$ and $d \neq c$.

Now, we construct the following matrix using row and column selection:

$$\tilde{\mathbf{F}}(\mathcal{I}, :)^{\dagger} \begin{bmatrix} [\mathbf{R}]_{i_1, j_1} / [\mathbf{Q}]_{i_1, j_1} \\ \vdots \\ [\mathbf{R}]_{i_T, j_1} / [\mathbf{Q}]_{i_T, j_1} \end{bmatrix} = \tilde{\mathbf{F}}(\mathcal{I}, :)^{\dagger} \mathbf{v}'_{j_1} \quad (20)$$

$$= \mathbf{J}(:, l) \quad (21)$$

where $i_l \in \mathcal{I}$ and $j_l \in \mathcal{J}$. We then obtain \mathbf{J} from

$$\mathbf{J}_{\mathcal{I}, \mathcal{J}} = \tilde{\mathbf{F}}(\mathcal{I}, :)^{\dagger} [\mathbf{v}'_{j_1} \dots \mathbf{v}'_{j_T}] \left(\tilde{\mathbf{F}}(\mathcal{J}, :)^{\dagger} \right)^H. \quad (22)$$

The dependency of \mathbf{J} on the index is explicitly shown above. As seen in (22), the channel covariance matrix can be obtained using a $T \times T$ sampled data covariance matrix with element-wise scaling by the precoding coefficient. The same procedure can be applied to the unused samples of the data covariance

²The equi-spacing selection is advantageous since it yields a Vandermonde matrix $\tilde{\mathbf{F}}(\mathcal{I}, :)$ with condition number of one.

matrix. In finite sample case, noise is not thoroughly averaged out at the off-diagonal elements of the sample data covariance matrix $\hat{\mathbf{R}} = \frac{1}{N_s} \sum_{i=1}^{N_s} \tilde{\mathbf{y}}_i \tilde{\mathbf{y}}_i^H$. Thus, the noise can further be averaged out by averaging $\mathbf{J}_{\mathcal{I},\mathcal{J}}$ over different index sets for better estimation:

$$\bar{\mathbf{J}} = \frac{1}{M} \sum_{\mathcal{I},\mathcal{J}} \mathbf{J}_{\mathcal{I},\mathcal{J}}, \quad (23)$$

where M is the number of all possible row and column selections. Then, the low rank decomposition can be applied to $\bar{\mathbf{J}}$ to obtain \mathbf{h}_k blindly.

B. Multi-User Case ($K > 1$)

In this subsection, we propose a different procedure for multi-user blind channel estimation based on our single user approach. The key difference from [4] is to use a separate precoding matrix for each user exploiting the degree of freedom in the precoder matrix size. This method provides complete separation of user channel. Here, we only consider two user case but the procedure is easily generalized to multi-user case. This situation occurs that a terminal station is located in the hand-over area of a cellular network and the terminal station receives signals from multiple basestations and all basestations fully transmit over all subcarriers with frequency reuse factor of one. In this case, the data covariance matrix is given by

$$\mathbf{R} = \underbrace{\tilde{\mathbf{h}}_1 \tilde{\mathbf{h}}_1^H}_{\bar{\mathbf{J}}_1} \odot \mathbf{Q}_1 + \underbrace{\tilde{\mathbf{h}}_2 \tilde{\mathbf{h}}_2^H}_{\bar{\mathbf{J}}_2} \odot \mathbf{Q}_2 + \sigma_n^2 \mathbf{I}. \quad (24)$$

Similarly to the single-user case, the elements of the covariance matrix are expressed by

$$\begin{aligned} [\mathbf{R}]_{i,i} &= |\tilde{h}_1(i)|^2 [\mathbf{Q}_1]_{i,i} + |\tilde{h}_2(i)|^2 [\mathbf{Q}_2]_{i,i} + \sigma_n^2, \\ [\mathbf{R}]_{i,j} &= \tilde{h}_1(i) \tilde{h}_1^*(j) [\mathbf{Q}_1]_{i,j} + \tilde{h}_2(i) \tilde{h}_2^*(j) [\mathbf{Q}_2]_{i,j}, \quad i \neq j. \end{aligned}$$

The complete separation between the two users is possible by designing \mathbf{Q}_1 and \mathbf{Q}_2 with non-overlapping non-zero positions. To this end, we consider a subset of precoding matrices composed of non-zero elements with equi-spacing in row and column positions and zeros for all other positions. We call such a matrix a zero-hole matrix here. Under the non-overlapping zero-hole precoding matrix design, we have for given $j_l \in \mathcal{J}$,

$$[\mathbf{R}]_{i_l, j_l} = \tilde{h}_1(i_l) \tilde{h}_1^*(j_l) [\mathbf{Q}_1]_{i_l, j_l}, \quad \text{for } i_l \in \mathcal{I}_1 \quad (25)$$

$$[\mathbf{R}]_{i'_l, j_l} = \tilde{h}_2(i'_l) \tilde{h}_2^*(j_l) [\mathbf{Q}_2]_{i'_l, j_l}, \quad \text{for } i'_l \in \mathcal{I}_2 \quad (26)$$

such that $\mathcal{I}_1 \triangleq \{i_l | [\mathbf{Q}_1]_{i_l, j} \neq 0, [\mathbf{Q}_2]_{i_l, j} = 0 \text{ and } i_l \in \mathcal{I}\}$ and $\mathcal{I}_2 \triangleq \{i'_l | [\mathbf{Q}_1]_{i'_l, j} = 0, [\mathbf{Q}_2]_{i'_l, j} \neq 0 \text{ and } i'_l \in \mathcal{I}\}$. Then, as in the single-user case $\bar{\mathbf{J}}_k$ can easily be obtained by

$$[\mathbf{R}]_{i_l, j_l} / [\mathbf{Q}]_{i_l, j_l} = \tilde{h}_1(i_l) \tilde{h}_1^*(j_l) [\mathbf{Q}_1]_{i_l, j_l} = \bar{\mathbf{J}}_1(i_l, j_l), \quad (27)$$

$$[\mathbf{R}]_{i'_l, j_l} / [\mathbf{Q}]_{i'_l, j_l} = \tilde{h}_2(i'_l) \tilde{h}_2^*(j_l) [\mathbf{Q}_2]_{i'_l, j_l} = \bar{\mathbf{J}}_2(i'_l, j_l). \quad (28)$$

Further averaging is possible for different index selection for the same user, and low rank decomposition is applied separately to each user's $\bar{\mathbf{J}}_k$ to obtain the channel \mathbf{h}_k .

IV. OPTIMAL PRECODER DESIGN

In this section, we consider optimal design for precoder under the zero-hole structure using the same MSE criterion as in [4]. However, we provide optimal solution under the non-overlapping zero-hole precoding scheme, whereas numerical search was performed for optimality under a simple precoder structure in [4]. It will be shown shortly in the next section that the optimal precoder here outperforms the previous method significantly at low SNR. Once the non-overlapping zero-hole precoding is applied to different users, each user's signal is separated and the optimal design is separate for each user. If we have perfect data covariance matrix \mathbf{R} , any design of precoder provides exact \mathbf{J}_k for true \mathbf{h}_k with scale ambiguity. Thus, the optimality of design is relevant only for the estimation using the imperfect sample data covariance matrix corrupted by noise and finite sample effect. The difference between the sample covariance and true covariance matrices is defined as

$$\Delta \mathbf{R} \triangleq \hat{\mathbf{R}} - \mathbf{R}, \quad (29)$$

and it is known from [4],[8] that

$$\mathbb{E} \left\{ |[\Delta \mathbf{R}]_{i_l, j_l}|^2 \right\} = \frac{1}{N_s} [\mathbf{R}]_{i_l, i_l} [\mathbf{R}]_{j_l, j_l}. \quad (30)$$

where N_s is the number of samples used to calculate the sample covariance matrix. Then, the difference between $\hat{\mathbf{v}}'_{j_l}$ and \mathbf{v}'_{j_l} is simply given from (10) by

$$\Delta \mathbf{v}'_{j_l} = \left[\frac{[\Delta \mathbf{R}]_{i_1, j_l}}{[\mathbf{Q}]_{i_1, j_l}}, \dots, \frac{[\Delta \mathbf{R}]_{i_T, j_l}}{[\mathbf{Q}]_{i_T, j_l}} \right]^T, \quad (31)$$

where $i_l \in \mathcal{I}$ and $j_l \in \mathcal{J}$. Based on (30) and (31), the estimation error of \mathbf{J}_k is given similarly to [4] by

$$\Delta \mathbf{J}_k = \tilde{\mathbf{F}}(\mathcal{I}, :)^{\dagger} [\Delta \mathbf{v}'_{j_1} \cdots \Delta \mathbf{v}'_{j_T}] \left(\tilde{\mathbf{F}}(\mathcal{J}, :)^{\dagger} \right)^H. \quad (32)$$

and this estimation error should be minimized since \mathbf{J}_k is the final statistic to obtain the channel \mathbf{h}_k .

A. Optimal Design

Assume that the precoder satisfy the following three conditions.

C.1 \mathbf{Q} is positive semi-definite.

C.2 $\text{tr}\{\mathbf{Q}\} = N$

C.3 \mathbf{Q} has a series of non-zero elements with equi-spacing, i.e., it is a zero-hole matrix.

Condition 1 is required in order for the precoder \mathbf{W}_k to be uniquely decomposed with a symmetric positive semidefinite square root [5]. Condition 2 guarantees that there is no power boosting. Thus, the optimization problem is formulated as

$$\min_{\mathbf{Q}} \quad \mathbb{E} \left\{ \|\Delta \mathbf{J}_k\|_F^2 \right\} \quad (33)$$

$$\text{s.t.} \quad \begin{cases} \mathbf{Q} \succeq 0 \\ \text{tr}(\mathbf{Q}) = N \\ \mathbf{Q} \text{ is a zero-hole matrix.} \end{cases} \quad (34)$$

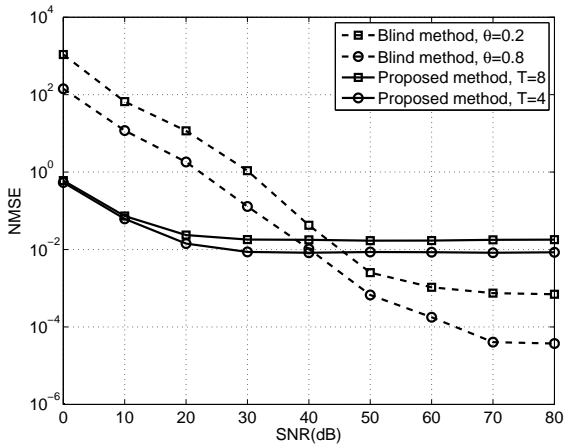


Fig. 2. NMSE versus SNR ($K = 1, N_s = 500, N_{MC} = 1000$)

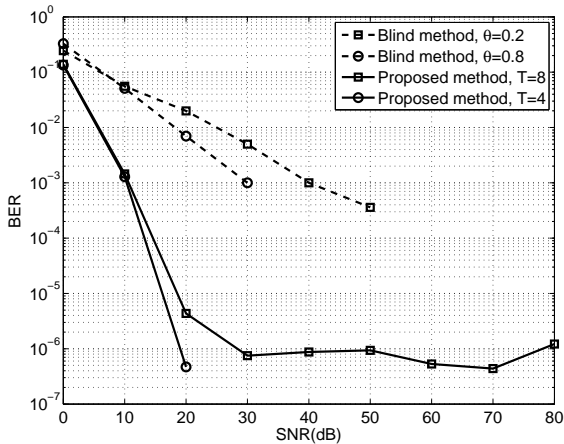


Fig. 3. BER versus SNR ($K = 1, N_s = 500, N_{MC} = 1000$)

where $\|\cdot\|_F$ denotes the Frobenius norm. Since this problem does not render easy solution, we restrict the constraint of the optimization further as follows.

C.1' \mathbf{Q} is diagonally dominant, i.e.,

$$|Q_{i,i}| \geq \sum_{j,j \neq i} |Q_{i,j}|. \quad (35)$$

C.2' The diagonal elements of \mathbf{Q} is one.

Note that C.1' and C.2' satisfy C.1 and C.2 and C.2' implies even power allocation.

Theorem 1: Under the constraints C.1', C.2' and C.3, $\mathbb{E}\{\|\Delta \mathbf{J}_k\|_F^2\}$ decreases monotonically as the number T of tones decreases with $T \geq L + 1$. Thus, the optimal solution to (33) occurs at $T = L + 1$.

Theorem 2: Under the constraints C.1', C.2' and C.3, $\mathbb{E}\{\|\Delta \mathbf{J}_k\|_F^2\}$ is minimized if the absolute values of the off-diagonal elements are identical.

(Proofs are omitted due to space limitation.)

V. NUMERICAL RESULTS

Here, we provide some numerical results for the proposed design method. Simulations are performed to estimate the

channel and the estimator performance is evaluated in terms of the normalized mean square error (NMSE) defined as

$$\text{NMSE} = \frac{1}{N_{MC}} \sum_{n=1}^{N_{MC}} \frac{\|\hat{\mathbf{h}}_n - \mathbf{h}\|_F^2}{\|\mathbf{h}\|_F^2}, \quad (36)$$

where $\hat{\mathbf{h}}_n$ denotes the n -th run estimate of \mathbf{h} and N_{MC} denotes the number of Monte Carlo runs.

The input symbols were independent and identically-distributed (i.i.d.) bi-phase shift keying (BPSK) symbols. Under the deterministic channel model, the true channel vector of three taps was used with channel coefficients $h(0) = 0.6127 + 0.9408j$, $h(1) = -0.9040 + 0.4294j$, and $h(2) = 0.7771 + 0.8671j$. Fig. 2 shows the estimation performance for the previous algorithm [4] and the proposed algorithm. It is seen that the proposed method outperforms the previous method at low SNR while the previous method is better in the high SNR. It is also seen that the estimation performance at $T = 4$ is better than that at $T = 8$. Fig. 3 shows the corresponding BER performance.

VI. CONCLUSION

We have considered the blind channel estimation for multiple single antenna OFDM systems coexisting in the same band. Based on the previous work using linear precoding for blind estimation, we have proposed a more efficient algorithm to separate user using non-overlapping precoder covariance at low SNR. We have derived an optimal design for such precoding scheme. Numerical results shows the validity of our analysis. The proposed method can be used to estimate the channels from multiple basestations to a single receiver even in case of full frequency reuse.

REFERENCES

- [1] H. Liu, G. Xu, L. Tong, and T. Kailath, "Recent developments in blind channel equalization: from cyclostationarity to subspaces," *IEEE Trans. Signal Processing*, vol. 50, pp. 83–99, Sep. 1996.
- [2] Y. Zeng and T. Ng, "A semi-blind channel estimation method for multiuser multiantenna OFDM systems," *IEEE Trans. Signal Processing*, vol. 52, no. 5, pp. 1419–1429, May 2004.
- [3] S. Yatawatta and P. Petropulu, "Blind channel estimation in MIMO OFDM systems with multiuser interference," *IEEE Trans. Signal Processing*, vol. 54, no. 3, pp. 1054–1068, Mar. 2006.
- [4] X. Liao, L. Fan, and F. Gao, "Blind channel estimation for OFDM modulated two-way relay network," *IEEE WCNC*, Sydney, Australia, Apr. 2010.
- [5] G. H. Golub and C. F. van Loan, *Matrix Computations*, Johns Hopkins University Press, Baltimore, Md, USA, 1989.
- [6] R. Negi and J. Coiffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. Consumer Electronics*, vol. 44, no. 3, pp. 1122–1128, Aug. 1998.
- [7] E. Moulines, P. Duhamel, J. Cardoso, and S. Mayrargue, "Subspace methods for the blind identification of multichannel FIR filters," *IEEE Trans. Signal Processing*, vol. 43, no. 2, pp. 516–525, Feb. 1995.
- [8] Z. Xu, "On the second-order statistics of the weighted sample covariance matrix," *IEEE Trans. Signal Processing*, vol. 51, no. 2, pp. 527–534, Feb. 2003.
- [9] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, New York, 1985.