# CHANNEL TRACKING FOR FAST FADING LONG-CODE WCDMA

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## ABSTRACT

A new technique for tracking of fast fading channels in long code CDMA is proposed. Using a linear interpolation model, the proposed method tracks the time-varying channel without pilot symbols or with sparsely inserted pilots. The proposed method can be implemented efficiently using a state-space inversion technique. A new identifiability condition is established and the performance of the method is assessed through the mean square error and bit error rate.

## 1. INTRODUCTION

Recent code division multiple access systems such as WCDMA adopted coherent detection schemes with pilot symbols in the reverse link to increase the system capacity. In such systems, channel estimation with a reasonable performance over a wide range of fading rates plays a crucial role.

Channel estimators based on the block fading model do not work satisfactorily for fast fading where channel changes rapidly within a time interval between pilot symbols; Additional pilot insertion is required with the conventional estimation methods. Several channel tracking methods are proposed based on time multiplexed pilot symbols and interpolation techniques e.g. [1] [8]. These methods utilize only pilot symbols over multiple slots, which requires pilot symbols with high SNR and long observation time. Others consider Wiener filter approaches which requires the knowledge of the Doppler spread and the signal-to-interference ratio [4].

In this paper, we present a new channel estimation technique which utilizes multipath structure and tracks fast fading channels effectively without the insertion of additional pilot symbols. Using a linear interpolation model, the proposed method estimates the channel coefficients at selected sampling points and tracks the channel for the whole slot. The proposed method can be implemented with a state-space inversion technique with a comparable amount of complexity with conventional RAKE receiver [5].

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The paper is organized as follows. The data model of a CDMA system is described in Section 2. A blind channel tracking method based on linear interpolation channel model is proposed in Section 3. In Section 4, the performance of the proposed method is assessed by Monte Carlo simulations and compared with the estimation with block fading model.

## 2. DATA MODEL

We consider an asynchronous CDMA system with $K$ users with long spreading sequences of spreading gain $G$ and slotted transmissions of $M$ symbol size. As illustrated in Fig.1, user $i$ symbol sequence $s_i(t)$ is scrambled with the long spreading code $c_i(t)$ and transmitted.

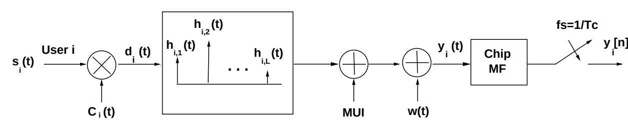


**Fig. 1**. System model

We assume that the channel of a particular user $i$ consists of $L$ independent multipaths each of which is a bandlimited deterministic waveform with bandwidth $f_D$ the maximum Doppler frequency [3]. Due to fast fading, we let the multipath coefficients vary from symbol to symbol while remaining constant over one symbol period $T_s$. The delay profile of multipaths is assumed to be invariant within one slot[1]. To simplify the model, we assume that all users are chiprate synchronized, that is, the delays between paths are multiples of chip interval $T_c$. Specifically, the continuous-time time-varying channel impulse response of user $i$ is given as

$$h_i(t,\tau) = \sum_{l=1}^{L} h_{il}(t)\delta(\tau - lT_c - d_iT_c), \qquad (1)$$

[1]Fast fading results mainly from the phase variation of carrier not from the delay changes

where $h_{il}(t)$ is the path coefficient waveform, $d_i$ the delay of user $i$ relative to the slot reference. We assume that $d_i$ and $L$ is known[2]. The received signal $y_i(t)$ is passed through a chip waveform matched filter and sampled synchronously at the chiprate. Since the channel is linear and has a finite impulse response, the noiseless chiprate output sample $y_i[n]$ corresponding to user $i$ is expressed as the convolution sum

$$y_i[n] = \sum_{l=1}^{L} d_i[n-l] h_{il}[n], \tag{2}$$

where $d_i[n] = c_i[n] s_i[\lfloor \frac{n}{G} \rfloor]$, $s_i[m]$ is the discrete-time symbol rate data sequence, and $c_i[n]$ the chiprate spreading sequence. The received noiseless signal vector $\mathbf{y}_{im}$ due to the $m$th symbol $s_{im}$ of user $i$ is given in a matrix form as

$$\mathbf{y}_{im} = \mathbf{T}_{im} \mathbf{h}_{im} s_{im}, \tag{3}$$

where $\mathbf{T}_{im}$ is the Toeplitz matrix whose first column is made of $mG + d_i$ zeros followed by vector $\mathbf{c}_{im} = [c_i[mG+1], \ldots, c_i[(m+1)G]]^T$ and additional zeros. Vector $\mathbf{h}_{im}$ is defined as

$$\mathbf{h}_{im} \triangleq \begin{bmatrix} h_{im}^{(1)} \\ h_{im}^{(2)} \\ \vdots \\ h_{im}^{(L)} \end{bmatrix} \triangleq \begin{bmatrix} h_{i1}(mT_s) \\ h_{i2}(mT_s) \\ \vdots \\ h_{iL}(mT_s) \end{bmatrix}. \tag{4}$$

The size of $\mathbf{y}_{im}$ is the total number of chips of the entire $M$-symbol slot. For user $i$, the total received noiseless signal is expressed as

$$\begin{aligned} \mathbf{y}_i &= \sum_{m=1}^{M} \mathbf{T}_{im} \mathbf{h}_{im} s_{im} \\ &= \mathbf{T}_i \text{diag}(\mathbf{h}_{i1}, \ldots, \mathbf{h}_{iM}) \mathbf{s}_i, \end{aligned} \tag{5}$$

where $\mathbf{s}_i = [s_{i1} \ldots s_{iM}]^T$ and $\mathbf{T}_i \triangleq [\mathbf{T}_{i1}, \cdots, \mathbf{T}_{iM}]$ which has a special structure of sparse block Toeplitz form which is exploited for efficient implementation of matrix inversion [5]. Including all users and noise, we have the multiuser data model as

$$\begin{aligned} \mathbf{y} &= \sum_{i=1}^{K} \mathbf{T}_i \text{diag}(\mathbf{h}_{i1}, \ldots, \mathbf{h}_{iM}) \mathbf{s}_i + \mathbf{w} \\ &= \mathbf{THs} + \mathbf{w}, \end{aligned} \tag{6}$$

where the overall code matrix $\mathbf{T} = [\mathbf{T}_1, \ldots, \mathbf{T}_K)]$, $\mathbf{H} = \text{diag}(\mathbf{h}_{11}, \ldots, \mathbf{h}_{1M}, \mathbf{h}_{21}, \ldots, \mathbf{h}_{2M}, \ldots \mathbf{h}_{KM})$, and $\mathbf{w}$ is additive Gaussian noise. We also assume the following

A1: The code matrix $\mathbf{T}$ is known and has full column rank.

A2: The noise vector is circularly symmetric complex Gaussian $\mathbf{w} \sim \mathcal{N}(\mathbf{0}, \sigma^2 \mathbf{I})$ with possibly unknown $\sigma^2$.

Assumption (A1) implies that the receiver knows codes of all users. This assumption is usually valid for the uplink and full column assumption is satisfied almost surely with reasonably high spreading factors.

## 3. FAST FADING CHANNEL ESTIMATION

### 3.1. Linear interpolation channel model

We consider a linear interpolation channel model under the deterministic parameter assumption. Among various linear interpolations, we consider the $N$-sample time domain approach[3] which includes a broad range of interpolation techniques such as piecewise linear, polynomial, ideal low pass interpolations. A similar modeling of a time-varying channel with a truncated basis was proposed in [7].

We assume that the channel at an arbitrary time within a slot is a linear combination of channels at sample points which are not necessarily the pilot positions. Consider the $l$ channel path of user $i$. The channel $h_{im}^{(l)}$ at $m$th symbol

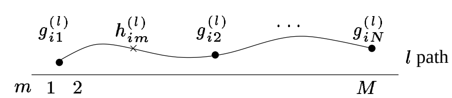


**Fig. 2**. $N$ sample interpolation model

interval is modeled as

$$h_{im}^{(l)} = \alpha_{m1} g_{i1}^{(l)} + \cdots + \alpha_{mN} g_{iN}^{(l)}, \; m = 1, \ldots, M, \tag{7}$$

where $\alpha_{mn}$ is the interpolation coefficient for symbol $m$ and sample $n$. Stacking all the multipaths corresponding to the same symbol and user, we have

$$\mathbf{h}_{im} = \begin{bmatrix} g_{i1}^{(1)} & g_{i2}^{(1)} & \cdots & g_{iN}^{(1)} \\ g_{i1}^{(2)} & g_{i2}^{(2)} & \cdots & g_{iN}^{(2)} \\ \vdots & & & \vdots \\ g_{i1}^{(L)} & g_{i2}^{(L)} & \cdots & g_{iN}^{(L)} \end{bmatrix} \begin{bmatrix} \alpha_{m1} \\ \alpha_{m2} \\ \vdots \\ \alpha_{mN} \end{bmatrix} \triangleq \mathbf{G}_i \mathbf{a}_m, \tag{8}$$

where $\mathbf{G}_i$ and $\mathbf{a}_m$ are defined correspondingly. The interpolation coefficient vector $\mathbf{a}_m$ is determined by the selected interpolation method and $\mathbf{G}_i$ is the unknown channel parameter which is invariant over a slot. We further assume the following

A3: The matrix $\mathbf{G}$ has full column rank.

[2] Rough timing knowledge is enough since we parameterize the delay cluster with sufficient number of fingers $L$

[3] Frequency domain approach can be also used with $N$ frequency components for bandlimited channels.

Assumption (A3) implies that the number of multipaths is larger than that of the sampling points which is usually valid due to the abundance of the multipaths in most mobile channels and several sample points are sufficient to track the channel change within a slot effectively. The design parameter $N$ is chosen considering the fading velocity so that the channel vectors at different sampling points are linearly independent almost surely. This suffices to the validity of the assumption (A3).

### 3.2. Blind multiuser channel estimation algorithm

We propose a blind channel estimator based on the linear interpolation channel model exploiting the multipath structure of channel. We assume that the channel and symbols are deterministic parameters.

The multiuser signals are separated by decorrelating or regularized least square front end. The required matrix inversion is efficiently implemented by an algorithm using the state-space technique[5][6]. The output of the decorrelator is given by

$$\begin{aligned} \mathbf{z} &= \mathbf{T}^{\dagger}\mathbf{y} \\ &= \mathrm{diag}(\mathbf{h}_{11}, \ldots, \mathbf{h}_{1M}, \mathbf{h}_{21}, \ldots, \mathbf{h}_{KM})\mathbf{s} + \mathbf{n}, \end{aligned}$$

where $(\cdot)^{\dagger}$ is pseudo inverse. Let $\mathbf{z}_{im}$ be the subvector of length $L$ corresponding to $m$th symbol of user $i$. Due to the diagonal structure of $\mathbf{H}$, the vector $\mathbf{z}_{im}$ us given by

$$\begin{aligned} \mathbf{z}_{im} &= \mathbf{h}_{im}s_{im} + \mathbf{n}_{im} \\ &= \mathbf{G}_i\mathbf{a}_m s_{im} + \mathbf{n}_{im}, \quad m = 1, \ldots, M, \end{aligned} \tag{9}$$

where the colored noise $\mathbf{n}_{im}$ has distribution of $\mathcal{N}(\mathbf{0}, \sigma^2\Sigma_{im})$, and the covariance $\Sigma_{im}$ is the $L \times L$ submatrix obtained from the $(i-1)M + m$th diagonal block of $\mathbf{T}^{\dagger}(\mathbf{T}^{\dagger})^H$.

#### 3.2.1. *Noise free case*

Consider the noise free case. With the deterministic assumption on $\mathbf{G}_i$, the column space of $\mathbf{G}_i$ is obtained by singular value decomposition

$$\mathbf{U}_i\Sigma_i\mathbf{V}_i^H = \mathbf{Z}_i \triangleq [\mathbf{z}_{i1}, \mathbf{z}_{i2}, \ldots, \mathbf{z}_{iM}]. \tag{10}$$

Then $\mathbf{G}_i$ is given as

$$\mathbf{G}_i = \mathbf{U}_i\mathbf{S}_i, \tag{11}$$

where $\mathbf{S}_i$ is a invertible $N \times N$ unknown square matrix from the assumption A(3). Projecting $\mathbf{z}_{im}$ to the column space of $\mathbf{U}_i$, we have the following square system

$$\mathbf{x}_{im} \triangleq \mathbf{U}_i^H\mathbf{z}_{im} = \mathbf{S}_i\mathbf{a}_m s_{im}, \;\; m = 1, \ldots, M. \tag{12}$$

Let $\mathbf{W}_i \triangleq \mathbf{S}_i^{-1}$ with the following row partition

$$\mathbf{W}_i = \begin{bmatrix} \mathbf{w}_{i1}^H \\ \mathbf{w}_{i2}^H \\ \vdots \\ \mathbf{w}_{iN}^H \end{bmatrix}. \tag{13}$$

Multiplying $\mathbf{x}_{im}$ by $\mathbf{W}_i$ from the left gives

$$\begin{bmatrix} \mathbf{w}_{i1}^H \\ \mathbf{w}_{i2}^H \\ \vdots \\ \mathbf{w}_{iN}^H \end{bmatrix} \mathbf{x}_{im} = \begin{bmatrix} \alpha_{m1} \\ \alpha_{m2} \\ \vdots \\ \alpha_{mN} \end{bmatrix} s_{im},$$

Assume $\alpha_{mn} \neq 0, \; n = 1, \cdots, N$ and $s_{im} \neq 0$ for given $m$, which is valid for most interpolation points and modulation schemes. Notice that multiplying row $j$,$k$ by $\alpha_{mk}$, $\alpha_{mj}$ respectively gives the same value $\alpha_{mj}\alpha_{mk}s_{im}$. Taking difference between two rows related to $m$th symbol data, a system of equations are obtained similarly as in [2]

$$\tilde{\mathbf{X}}_{im}\mathbf{w}_i = \mathbf{0} \tag{14}$$

where $\mathbf{w}_i \triangleq [\mathbf{w}_{i1}^H, \cdots, \mathbf{w}_{iN}^H]^H$ and

$$\tilde{\mathbf{X}}_{im} \triangleq \begin{bmatrix} \tilde{\mathbf{x}}_{im1}^H & -\tilde{\mathbf{x}}_{im2}^H & 0 & \cdots & \cdots & 0 \\ \tilde{\mathbf{x}}_{im1}^H & 0 & -\tilde{\mathbf{x}}_{im3}^H & & & \vdots \\ \vdots & 0 & 0 & \ddots & & \vdots \\ \vdots & & & & \ddots & 0 \\ \tilde{\mathbf{x}}_{im1}^H & 0 & \cdots & \cdots & 0 & -\tilde{\mathbf{x}}_{imN}^H \\ 0 & \tilde{\mathbf{x}}_{im2}^H & -\tilde{\mathbf{x}}_{im3}^H & 0 & \cdots & 0 \\ 0 & \vdots & 0 & \ddots & \ddots & \vdots \\ 0 & \tilde{\mathbf{x}}_{im2}^H & 0 & \cdots & 0 & -\tilde{\mathbf{x}}_{imN}^H \\ 0 & 0 & \tilde{\mathbf{x}}_{im3}^H & -\tilde{\mathbf{x}}_{im4}^H & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \tilde{\mathbf{x}}_{im,N-1}^H & -\tilde{\mathbf{x}}_{imN}^H \end{bmatrix}$$

and $\tilde{\mathbf{x}}_{imj} \triangleq \alpha_{mk}\mathbf{x}_{im}$ and $\tilde{\mathbf{x}}_{imk} \triangleq \alpha_{mj}\mathbf{x}_{im}$ for given $(j, k)$ pair. Combining all the symbol data gives

$$\mathbf{X}_i\mathbf{w}_i = \mathbf{0} \tag{15}$$

where $\mathbf{X}_i \triangleq [\tilde{\mathbf{X}}_{i1}^H, \cdots, \tilde{\mathbf{X}}_{iM}^H]^H$.

**Proposition 1 (Identifiability)** *The $MN(N-1)/2 \times N^2$ matrix $\mathbf{X}_i$ is a matrix with rank $N^2 - 1$, i.e. the column rank is deficient by one. Hence, $\mathbf{w}_i$ is the unique null space of $\mathbf{X}_i$ and is blindly identifiable up to a scale factor.*

Hence,$\mathbf{G}_i$ is identifiable by eq.(11).

## 4. NUMERICAL RESULTS

The performance of the proposed method focused on the algorithm itself is evaluated first though the Cramér-Rao bound and Monte Carlo simulation using a channel generated by the interpolation model in section 3.1 for a single user case. To exclude the channel modeling error, the channel is generated according to the interpolation model with three sample points and sinc interpolation coefficients. The scrambling code is generated randomly with spreading factor $G = 16$ and slot length $M = 160$ symbols. The number of multipaths $L$ is 4 with equal average magnitude. The signal-to-noise ratio is defined as $\frac{Avg(||\mathbf{h}_{im}||^2)G}{\sigma^2}$. The scale ambiguity is resolved using one pilot symbol placed at the left end of slot. As shown in fig. 3, the method shows a good mean square error performance and almost reaches the the Cramér-Rao bound at medium and high SNR.

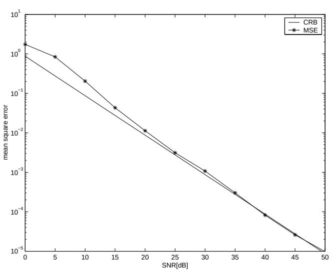


**Fig. 3**. Mean square error:interpolation channel

The performance of the proposed method is evaluated for a bandlimited waveform channel. Since we assume the channel is deterministic, the channel waveform is generated with Jakes's model[3] with $f_D T_{slot} = 0.75$ and truncated for one slot length. Two equal power asynchronous users with delay of a half symbol interval are simulated. Other parameters are the same as in the interpolation channel case. Fig. 4 shows the MSE performance of the proposed method with various interpolation techniques with three sample points. The proposed algorithm improves MSE performance much over the estimation using the pilot symbol and the block fading model. However, the proposed method also shows a performance floor at high SNR due to the imperfect modeling of the actual channel. Figure 5 shows the average bit error rate performance of a whitened RAKE receiver with the estimated channel. The increase of BER with respect to SNR for the estimation with block fading model shows that the additive noise works beneficially for detection since the estimation gives adverse values due to lack of tracking capability.

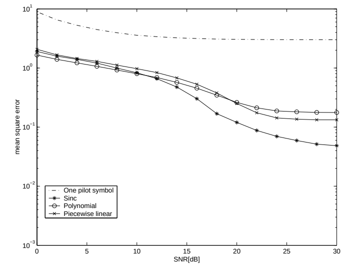


**Fig. 4**. Mean square error: low-pass channel

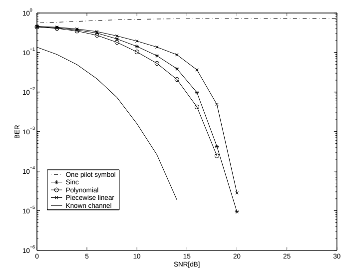


**Fig. 5**. Bit error rate: low-pass channel

## 5. CONCLUSION

We propose a new blind channel estimation technique which effectively tracks fastfading channels in long code CDMA systems and a new blind identifiability is established. Exploiting the multipath structure and interpolation model, the proposed method shows a significant improvement over the channel estimation with block fading model without insertion of additional pilot symbols.

## 6. REFERENCES

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