2005 Conference on Information Sciences and Systems, The Johns Hopkins University, March 16–18, 2005

Route Selection for Detection of Correlated Random Fields in Large Sensor Networks

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Abstract —

The problem of combining task performance with network-layer routing for the detection of correlated random fields is considered. Under the assumption of the Gauss-Markov structure along a given route, an optimal data aggregation algorithm is provided, and a performance metric that captures the detection performance is derived. It is shown that the logarithm of innovation variance is a reasonable choice for a link metric at high signal-to-noise ratio. An explicit formula for the metric as a function of the link length is also derived. The proposed metric can be used to select an optimal route yielding the best detection performance.

I. Introduction

Conventional multi-hop wireless *ad hoc* networks focus on the communication between nodes. The routing in such networks is primarily based on link metrics such as the hop count, minimum delay, traffic amount, etc. For energy-limited sensor networks different metrics such as the battery power of nodes, necessary transmission power between neighboring nodes have also been considered to distribute routes evenly over the network and enlarge the lifetime of networks [1, 2, 3, 4]. However, the main purpose of sensor networks lies in specific applications such as detection, monitoring, tracking, etc., using collaborative processing between sensor nodes. Hence, it is desirable to incorporate the performance of these tasks into routing in sensor networks. Examples of cross-layer approaches can be found in [5, 6, 7, 8, 9]. In particular, in [9] the authors investigated the effect of spatial correlation between signal sources on routing and data compression via the total energy consumption in the network to transfer overall information to a gateway node.

In this paper, we consider a cross-layer approach to combining the task performance at the application layer with routing at the network layer for detection applications in large-scale sensor networks. Specifically, we consider the detection of a spatially correlated random field using sensors deployed over a geographical region, where each sensor on a route to a gateway node receives the data from a neighboring sensor and also makes its own measurement of the phenomenon of interest at its location. The assumption of spatial correlation is appropriate for large sensor networks, especially for densely deployed ones. The spatial correlation can affect the performance of detection based on different routes significantly. That is, different routes result in distinct detection performances at the gateway node even if the number of sensors on routes is the same. (For commonly assumed independent and identically distributed (i.i.d.) signal fields, however, the geometry of routes to the gateway node is irrelevant and the performance depends only on the number of sensors or observations on a route.) Thus, for correlated signal cases it is natural to ask "which route gives the best detection performance, and what are the factors of a route that affect the detection performance?"

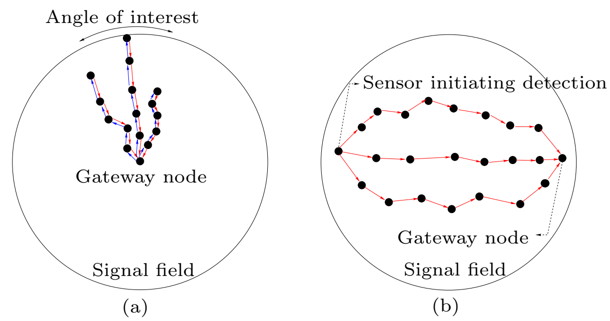

Figure 1: Detection and route selection: (a) Gateway node initiated detection, (b) sensor initiated detection.

I.A Summary of Results

We present a framework of combining the route selection and detection performance for multi-hop sensor networks. Our focus and contribution in this paper are twofold. First, we derive an optimal data aggregation algorithm for the detection of correlated random fields based on the location of sensors along a fusion route. Second, we obtain a link metric that captures the detection performance for a given route. To this goal, we simplify the correlation structure of signal fields of interest using the Gauss-Markov model and choose the error probability of the optimal detection based on the observation along a route as the performance criterion. Since the exact error probability of the optimal detection of correlated Gaussian signals is not available, we apply the Chernoff bound [10] that provides a tight upper bound on the detection error probability for large sample cases. However, the conventional expression of the Chernoff bound is given in terms of the eigenvalues of

*This work was supported in part by the Multidisciplinary University Research Initiative (MURI) under the Office of Naval Research Contract N00014-00-1-0564. Prepared through collaborative participation in the Communications and Networks Consortium sponsored by the U. S. Army Research Laboratory under the Collaborative Technology Alliance Program, Cooperative Agreement DAAD19-01-2-0011.

the covariance matrix of signal samples [11], which does not allow the decomposition of the overall performance into a sum of the incremental performance gain at each link.

Our approach to this problem is to exploit the expression of log-likelihood via the innovations representation [12]. By expressing the log-likelihood ratio in terms of the innovation at each link of a fusion route, we obtain an additive incremental cost at each link for the error bound. It is shown that the *logarithm of the innovation variance* at each link is a reasonable choice as a link metric to select an optimal route that gives the best detection performance at high signal-to-noise ratio (SNR). This metric can be easily calculated using the field diffusion coefficient and sensor locations. We also provide an explicit formula for the proposed metric as a function of the link length.

The remainder of the paper is organized as follows. In Section II we describe the signal model and a location-based optimal fusion algorithm. In Section III we derive a new link metric for the detection performance. Some discussion is presented about the results in Section IV, followed by the conclusion in Section V.

II. System Model and Optimal Data Aggregation

We consider the detection of a correlated random field $\mathcal{S}$ over a two-dimensional space $\mathcal{X}$ using sensors deployed over $\mathcal{X}$ under the Bayesian formulation, where the hypotheses H_0 and H_1 represent the event of no signal and the presence of signal $\mathcal{S}$ over $\mathcal{X}$, respectively. We assume that the signal field is static during the time period of observation and processing. We also assume that each sensor knows its own location and sensor observations are delivered to a gateway node via multihop routes. Since sensors are located within the signal field, each sensor on a route to the gateway node not only transfers data from the previous sensor but also makes its own observation (corrupted by the measurement noise) and delivers the aggregated data to the next sensor on the route. Thus, data fusion occurs along the route, and the final decision is made at the gateway node.

Suppose that a fusion route traverses sensor nodes N_1, N_2, $\cdots$, N_n where N_n is the gateway node. Let $\mathcal{R}(N_1,\cdots,N_n)$ denote this fusion route. To make further development tractable, we make the following assumption on the correlation structure of the random field $\mathcal{S}$.

A 1 *For any open simple route $\mathcal{R}$ traversing an arbitrary set of nodes contained in $\mathcal{X}$, the signal along $\mathcal{R}$ forms an one-dimensional stationary Gauss-Markov process, and the signal model is given by*

$$\frac{ds(l)}{dl} = -As(l) + Bu(l), \quad 0 \le l \le |\mathcal{R}|, \tag{1}$$

where $|\mathcal{R}|$ denotes the length of the route $\mathcal{R}$, diffusion coefficients $A \ge 0$ and B are known, and the initial condition is given by $s(0)$ which has Gaussian distribution $\mathcal{N}(0,\Pi_0)$ with $\Pi_0 = \frac{B^2}{2A}$. The process noise $u(l)$ is zero-mean white Gaussian with unit variance, independent of both $s(0)$ and the measurement noise of sensor.

Here, A represents the diffusion rate of the signal field with respect to distance. Assumption 1 may be an oversimplification for general correlation and curve shapes. However, it is reasonable for a class of curves that are almost straight contained in a homogeneous Gauss-Markov field.

II.A Optimal data fusion

In this section we present a location-based optimal data fusion algorithm along a given route under Assumption 1. Suppose that we have a fusion route $\mathcal{R}(N_1,\cdots,N_n)$ and the location of the n sensor nodes along the route is given by $\{\mathbf{x}_i, i=1,\cdots,n\}$. Then, the hypotheses for the observations along the route are given by

$$\begin{array}{lcl} H_0 & : & y_i = w_i, \\ H_1 & : & y_i = s_i + w_i, \quad i = 1,2,\cdots,n, \end{array} \tag{2}$$

where y_i is the observation at node N_i, $s_i \triangleq s(\mathbf{x}_i)$, and $\{w_i\}$ are i.i.d. sensor measurement noises from $\mathcal{N}(0,\sigma^2)$ with a known variance σ^2. The prior probabilities of H_0 and H_1 are given by π_0 and π_1, respectively. Under Assumption 1 the dynamics of signal sample s_i at node N_i are described by the following state-space model:

$$\begin{aligned} s_{i+1} &= a_i s_i + u_i, \\ a_i &= e^{-A\Delta_i}, \\ u_i &\sim \mathcal{N}(0, \Pi_0(1-a_i^2)), \end{aligned} \tag{3}$$

where the distance between two neighboring sensors on the route $\Delta_i = ||\mathbf{x}_{i+1} - \mathbf{x}_i||$. Due to the stationarity of the field, the variance of s_i is Π_0 for all i, and the SNR† for the observations is given by

$$\text{SNR} = \frac{\Pi_0}{\sigma^2}. \tag{4}$$

The optimal detector for (2) is given by a likelihood ratio detector:

$$\delta_B(y_1^n) = \begin{cases} H_1, & T \triangleq \log \frac{p_1(y_1^n)}{p_0(y_1^n)} \ge \tau \triangleq \log \frac{\pi_0}{\pi_1}, \\ H_0, & \text{o.w.,} \end{cases} \tag{5}$$

where $y_1^i \triangleq \{y_1, y_2, \cdots, y_i\}$ and $p_j(y_1^n), j = 0,1$, is the joint probability density of y_1^n under hypothesis H_j. The likelihood under H_0 is simply given by

$$p_0(y_1^n) = \frac{1}{(2\pi\sigma^2)^{n/2}} e^{-\frac{1}{2\sigma^2}\sum_{i=1}^n y_i^2}. \tag{6}$$

Given the signal evolution (3), the log-likelihood under H_1 is easily calculated by the Kalman recursion along the route [12], which eliminates the necessity of delivering all the observations to the gateway node. The well-known data fusion using the Kalman recursion can be readily combined with the location information of sensors.

Define $l_i \triangleq \log p_1(y_1^i)$. Then, we can decompose l_i using

$$p_1(y_1^i) = p_1(y_1^{i-1}) p_1(y_i | y_1^{i-1}). \tag{7}$$

Hence, we have

$$l_i = l_{i-1} + \log p_1(y_i | y_1^{i-1}). \tag{8}$$

Since the joint distribution of $\{y_1, y_2, \cdots, y_i\}$ is Gaussian from Assumption 1, the conditional distribution $p_1(y_i|y_1^{i-1})$ is also Gaussian with mean $\mathbb{E}_1\{y_i|y_1^{i-1}\}$ and variance $R_{e,i}$. The log-likelihood l_i upto the ith observation along the route is expressed using the innovations representation by

$$l_i = l_{i-1} - \frac{1}{2}\log(2\pi R_{e,i}) - \frac{1}{2}\frac{e_i^2}{R_{e,i}}, \tag{9}$$

†Note that here the SNR is the observation SNR not the SNR of the communication signal to the receiver noise.

where the innovation is given by $e_i \triangleq y_i - \hat{y}_{i|i-1}$ with variance $R_{e,i} = \mathbb{E}_1\{e_i^2\}$ and the minimum mean square error (MMSE) prediction $\hat{y}_{i|i-1}$ of y_i given y_1^{i-1} is the conditional expectation $\mathbb{E}_1\{y_i|y_1^{i-1}\}$. Thus, the log-likelihood under H_1 is given by [12]

$$\log p_1(\mathbf{y}_n) = l_n = -\frac{1}{2}\sum_{i=1}^{n}\log(2\pi R_{e,i}) - \frac{1}{2}\sum_{i=1}^{n}\frac{e_i^2}{R_{e,i}}. \quad (10)$$

Eq. (9) provides a recursive structure of a location-based fusion algorithm for the detection by incorporating the state-space model (3). The algorithm is described by the following steps.

Algorithm 1 (Location-based optimal data fusion)

1. Initialization at one end of a route.
   - $\hat{s}_{1|0} = 0$, $P_{1|0} = \Pi_0$, $l_0 = 0$.
2. Update at each sensor.
   - Calculation at current sensor.

$$\begin{aligned} e_i &= y_i - \hat{s}_{i|i-1}, \\ R_{e,i} &= P_{i|i-1} + \sigma^2, \quad (11) \\ l_i &= l_{i-1} - \frac{1}{2}\left(\log(2\pi R_{e,i}) + \frac{e_i^2}{R_{e,i}}\right). \end{aligned}$$

   - Incorporation of location information.

$$\begin{aligned} \Delta_i &= ||\mathbf{x}_{i+1} - \mathbf{x}_i||, \quad (12) \\ a_i &= e^{-A\Delta_i}, \quad Q_i = \Pi_0(1-a_i^2). \quad (13) \end{aligned}$$

   - Prediction for next sensor.

$$\begin{aligned} K_{p,i} &= (a_i P_{i|i-1})/R_{e,i}, \quad (14) \\ \hat{s}_{i+1|i} &= a_i \hat{s}_{i|i-1} + K_{p,i} e_i, \\ P_{i+1|i} &= a_i^2 P_{i|i-1} + Q_i - K_{p,i}^2 R_{e,i}. \quad (15) \end{aligned}$$

Since we assume that each sensor knows its own location, the only other necessary location information for sensor N_i is that of the next sensor for the prediction step (12). The information flow between nodes along the route is illustrated in Figure 2. (For the likelihood under H_0 the data fusion is a simple accumulation requiring only observation values.) Node N_i needs to receive the log-likelihood l_{i-1}, the prediction for itself $\hat{s}_{i|i-1}$ and the error covariance $P_{i|i-1}$ from node N_{i-1}, and the location information $\mathbf{x}_{i+1}$ from N_{i+1}.

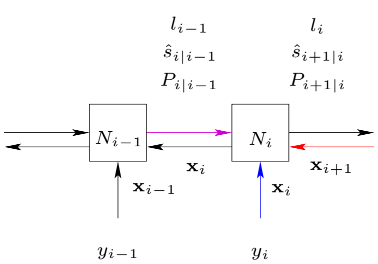

Figure 2: Information flow along a fusion route.

III. Performance Metric

In this section we address the problem of selecting an optimal route and propose a link metric that captures the detection performance for a fusion route. Since we are interested in the detection performance, it is natural to consider as a figure of merit the average error probability of the optimal detection for a given route. However, the exact calculation of error probability is not available for general Gauss-Markov signals [11]. Hence, we use the Chernoff bound [10] on the error probability as our performance criterion. The derived performance metric is designed so as to minimize the Chernoff (upper) bound on the average error probability. We show that at reasonably high SNR the logarithm of the normalized innovation variance for each link can be used as a link metric that captures the detection performance of a fusion route.

III.A Derivation

Consider a fusion route $\mathcal{R}(N_1, \cdots, N_n)$. Let us define the vector of observations along the route:

$$\mathbf{y}_n \triangleq [y_1, \cdots, y_n]^T. \quad (16)$$

Due to the Gauss-Markov assumption along the route, the distribution of $\mathbf{y}_n$ is given by

$$\mathbf{y}_n \sim \begin{cases} \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I}) & \text{under } H_0, \\ \mathcal{N}(\mathbf{0}, \mathbf{\Sigma}_s + \sigma^2\mathbf{I}) & \text{under } H_1, \end{cases} \quad (17)$$

where $\mathbf{I}$ is an identity matrix and $\mathbf{\Sigma}_s$ is the covariance matrix of the signal samples of the n nodes along the route. The Chernoff bound on the average error probability of the MAP detector (5) is given by [11]

$$P_e = \pi_0 P(\mathcal{E}|H_0) + \pi_1 P(\mathcal{E}|H_1) \leq \pi_0^{1-s}\pi_1^s e^{\mu_{T,0}(s)}, \quad (18)$$

where $\mu_{T,0}$ is the cumulant generating function of the log-likelihood ratio T in (5) under H_0, i.e.,

$$\mu_{T,0}(s) = \log \mathbb{E}_0\left\{e^{s\log\frac{p_1(y_1^n)}{p_0(y_1^n)}}\right\}, \quad 0 \leq s \leq 1. \quad (19)$$

The Chernoff information between $p_0(y_1^n)$ and $p_1(y_1^n)$ is defined as the exponent in (18) yielding the tightest bound, i.e.,

$$C\left(p_0(y_1^n), p_1(y_1^n)\right) \triangleq \sup_{0\leq s\leq 1}\{-\mu_{T,0}(s)\} \quad (20)$$

Thus, the desirable properties of a performance metric in terms of the proposed criterion are summarized as follows:

(i) The performance metric should capture the Chernoff information provided by the route.

(ii) The overall metric is represented as a sum of the contribution of each link to the performance.

The conventional procedure for the calculation of (19) involves a quadratic form of the observation vector (16) and the eigenvalues of $\mathbf{\Sigma}_s$ in (17). However, this approach does not allow us to decompose the error bound into a sum of the contribution of each link. Our approach is based on the innovations representation of the log-likelihood in the previous section. Using the innovations representation (6, 10), we have

$$\begin{aligned} \mu_{T,0}(s) &= \log \mathbb{E}_0\left\{\exp\left[s\left(-\frac{1}{2}\sum_{i=1}^{n}\log R_{e,i} - \frac{1}{2}\sum_{i=1}^{n}\frac{e_i^2}{R_{e,i}} \right.\right.\right. \quad (21) \\ &\qquad \left.\left.\left. + \frac{n}{2}\log\sigma^2 + \frac{1}{2}\sum_{i=1}^{n}\frac{y_i^2}{\sigma^2}\right)\right]\right\}, \\ &= s\left(-\frac{1}{2}\sum_{i=1}^{n}\log R_{e,i} + \frac{n}{2}\log\sigma^2\right) + \quad (22) \\ &\qquad \log \mathbb{E}_0\left\{\exp\left[s\left(-\frac{1}{2}\sum_{i=1}^{n}\frac{e_i^2}{R_{e,i}} + \frac{1}{2}\sum_{i=1}^{n}\frac{y_i^2}{\sigma^2}\right)\right]\right\}, \end{aligned}$$

where the second equality results from the fact that the innovation variance $R_{e,i}$ is deterministic. Suppose that the number n of hops along the route is sufficiently large. Then, $\frac{1}{n}\sum_{i=1}^{n} y_i^2$ converges almost surely to its mean $\sigma^2 (= \mathbb{E}_0\{y_i^2\})$ under H_0 by the strong law of large numbers (SLLN). Hence, for a route with many hops the second term in the argument of the exponential function in (22) can be approximated by $\frac{n}{2}$ which does not depend on the geometry of the route. Thus, for large n the cumulant generating function is approximated by

$$\begin{aligned}\mu_{T,0}(s) &\approx s\left(-\frac{1}{2}\sum_{i=1}^{n}\log R_{e,i} + \frac{n}{2}(\log\sigma^2+1)\right)\\ &\quad + \log \mathbb{E}_0\left\{\exp\left[s\left(-\frac{1}{2}\sum_{i=1}^{n}\frac{e_i^2}{R_{e,i}}\right)\right]\right\}. \qquad (23)\end{aligned}$$

Now consider the argument of the exponential function in (23). If the expectation were taken under H_1, then $\{e_i\}$ would form an independent sequence (true innovations sequence) and the sum of e_i^2 normalized by $R_{e,i}$ would be approximated by $\frac{n}{2}$ as well by the SLLN for a sufficiently large n. However, the expectation here is taken under H_0 where the observation sequence $\{y_i\}$ is an i.i.d. sequence of sensor measurement noises. In this case, the innovation e_i is represented by an output of a recursive linear filter, i.e., whitening filter, driven by the i.i.d. noise sequence with variance σ^2. Hence, the variance of e_i decreases to zero as the SNR increases (i.e., $\sigma^2 \to 0$). On the other hand, note that $R_{e,i}$ is the variance of the true innovation, i.e., e_i under H_1, and from (11) we have

$$R_{e,i} \to P_{i|i-1} \qquad (24)$$

as the SNR increases. As the SNR increases, the prediction error covariance $P_{i|i-1}$ converges to a positive constant for a transition coefficient a_i strictly less than one. Thus, $\frac{e_i^2}{R_{e,i}}$ converges to zero in mean square as the SNR increases, and the term is negligible compared with $\log R_{e,i}$. Hence, at high SNR the cumulant generating function is approximated by

$$\mu_{T,0}(s) \approx s\left(-\frac{1}{2}\sum_{i=1}^{n}\log R_{e,i} + \frac{n}{2}(\log\sigma^2+1)\right). \qquad (25)$$

Combining (18) and (25) yields

$$\begin{aligned}P_e \le B_c &\approx \pi_0^{1-s}\pi_1^{s} e^{-\frac{s}{2}\left\{\sum_{i=0}^{n-1}\log R_{e,i} - n(\log\sigma^2+1)\right\}}, \qquad (26)\\ &= \pi_0^{1-s}\pi_1^{s} e^{-\frac{s}{2}\left\{\sum_{i=0}^{n-1}\left(\log\frac{R_{e,i}}{\sigma^2}-1\right)\right\}}, \qquad (27)\end{aligned}$$

where $0 \le s \le 1$. Observing (26, 27), we recognize several facts. First, the optimization over the variable s $(0 \le s \le 1)$ to obtain the tightest bound is separable from the performance dependence on the route topology at high SNR. This is because, at high SNR, $\log\frac{R_{e,i}}{\sigma^2} > 1$ (i.e., $R_{e,i} >> \sigma^2$) and $R_{e,i}$ depends only on the SNR and route topology. Hence, the route selection can be done by considering only the route topology. In addition, the Chernoff information is attained at $s = 1$ and is approximately given by

$$C\left(p_0(y_1^n), p_1(y_1^n)\right) \approx \frac{1}{2}\sum_{i=0}^{n-1}\left(\log\frac{R_{e,i}}{\sigma^2} - 1\right) \approx \frac{1}{2}\sum_{i=0}^{n-1}\log\frac{R_{e,i}}{\sigma^2}. \qquad (28)$$

Second, we realize what constitutes the error performance of a route at high SNR. As seen in (26), there are positive and negative factors to the detection performance and both are deterministic quantities that do not depend on specific values of observations. For every hop along the route there is a performance degradation due to the measurement noise added by the sensor node, which is given by $(\log\sigma^2+1)$, and this degradation does not depend on the route topology. This follows our intuition since the degrading effect is caused by the measurement noise and the measurement noise at each sensor is i.i.d. The other factor is the performance improvement at each link which is given by the logarithm of the innovation variance at the link, $\log R_{e,i}$. This improvement factor is greater than the degradation factor since $R_{e,i} \gg \sigma^2$ at high SNR. Notice that the amount of performance improvement at each hop is not equal! It depends on how much new information (innovation) about the signal field each link provides to the detector; a link with larger $R_{e,i}$ provides a larger benefit. Hence, different routes with the same number of hops can provide different detection performances.

Expressing the bound by (27) gives a simple interpretation of the link metric. Define the normalized innovation variance as

$$r_{e,i} \triangleq \frac{R_{e,i}}{\sigma^2}. \qquad (29)$$

At high SNR the Chernoff information provided by a fusion route is approximated by a sum of the logarithm of the normalized innovation variance at each link. Since e_i has Gaussian distribution $\mathcal{N}(0, R_{e,i})$, the entropy of the innovation e_i at link i is given by $\frac{1}{2}\log(2\pi e R_{e,i})$. Hence, the overall metric in (28) is also interpreted as the accumulated entropy of the innovation process along the route.

III.B Link metric as a function of the link length

In this section, we derive an explicit formula for the proposed metric $\log r_{e,i}$ as a function of the link length.

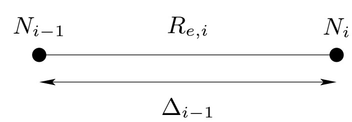

Figure 3: Link i.

Let us consider the innovation over link i given the previous links. From the Kalman recursion (11, 14, 15) we have a recursion for the MMSE prediction error:

$$P_{i|i-1} = \frac{\sigma^2 a_{i-1}^2 P_{i-1|i-2}}{P_{i-1|i-2}+\sigma^2} + Q_i, \qquad (30)$$

where $P_{i|i-1}$ is the error variance of the MMSE prediction for s_i given $\{y_0, \cdots, y_{i-1}\}$. Since $P_{i-1|i-2}$ depends only on the previous links, $\frac{P_{i-1|i-2}}{P_{i-1|i-2}+\sigma^2} =: K_{i-1}$ $(0 \le K_{i-1} \le 1)$ is a constant with respect to Δ_{i-1}. Thus, the innovation variance at link i is given by

$$\begin{aligned}R_{e,i} &= P_{i|i-1} + \sigma^2 = \sigma^2 a_{i-1}^2 \frac{P_{i-1|i-2}}{P_{i-1|i-2}+\sigma^2} + Q_{i-1} + \sigma^2,\\ &= \Pi_0 + \sigma^2 - (\Pi_0 - \sigma^2 K_{i-1})e^{-2A\Delta_{i-1}}, \qquad (31)\end{aligned}$$

where (31) is obtained by substituting $a_{i-1} = e^{-A\Delta_{i-1}}$ and $Q_{i-1} = \Pi_0(1-e^{-2A\Delta_{i-1}})$. Thus, the metric for link i is given by

$$\frac{1}{2}\log r_{e,i} = \frac{1}{2}\log\left\{\mathrm{SNR} + 1 - (\mathrm{SNR} - K_{i-1})e^{-2A\Delta_{i-1}}\right\}. \qquad (32)$$

Furthermore, at high SNR we have $K_{i-1} \approx 1$, and the proposed metric is approximated by

$$\frac{1}{2}\log r_{e,i} \approx \frac{1}{2}\log\left\{\mathrm{SNR} + 1 - (\mathrm{SNR}-1)e^{-2A\Delta_{i-1}}\right\}. \quad (33)$$

Figure 4 shows the link metric (33) as a function of the link length Δ_{i-1}. The figure shows several interesting properties of the link metric as a function of the link length. It is seen that the link metric increases as the link length Δ_{i-1} increases but eventually converges to $\frac{1}{2}\log(\mathrm{SNR}+1)$. In the previous

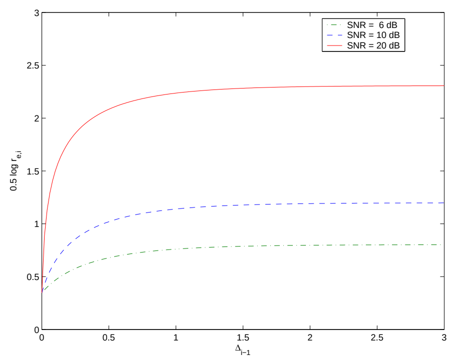

Figure 4: Link metric $\frac{1}{2}\log r_{e,i}$ versus Δ_i ($A = 1$).

sections, we see qualitatively that a longer link provides a greater contribution to the detection performance since $R_{e,i}$ increases monotonically as the link length increases. It is, however, seen that after a certain point of the link length the performance gain due to the increase of the link length is negligible. This reveals that a too long hop is not efficient for the performance considering the transmission energy required for the link since the required transmission energy increases with a polynomial order of the link length.

IV. Discussion

In this section, we discuss applications of the results in Section III and related issues .

First, we can find a direct application in gateway node initiated detection based on conventional on-demand *ad hoc* routings such as the dynamic source routing [13] (DSR) and the *ad hoc* on-demand distance vector routing [14] (AODV), as illustrated in Figure 1 (a). We can modify easily the existing reactive routing protocols to incorporate Algorithm 1. Especially, the DSR is well-suited to the modification.

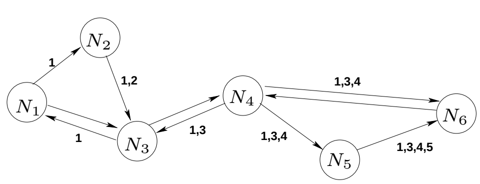

Figure 5: Dynamic source routing (DSR) [13].

In the combined algorithm that we propose, the gateway node first determines the direction of investigation and the number of hops (i.e., the number of observation samples) that is necessary to satisfy a certain detection performance, and initiates the detection operation by sending a message containing the decided number of hops and direction. In addition to the conventional DSR information we add the location information of each sensor in the downward propagation. This message propagates through the sensor network according to the DSR algorithm, as shown in Figure 5. When a node within the direction receives the detection initiation message and recognizes that the hop count up to itself matches the predetermined hop count in the message, it stops the message propagation and initiates Algorithm 1 along the established reverse path back to the gateway node carrying the sufficient statistic and the sum of $\log r_{e,i}$. In this case, the route between the gateway node to a node that satisfies the hop count determined by the DSR algorithm is the shortest hop route between them and is approximately a straight line in a 2-D space. Hence, Assumption 1 is reasonable for homogenous 2-D Gauss-Markov fields in this application. Since there may be many nodes that satisfy the hop count within the direction of interest, the gateway node may receive multiple responses to the initiated routing/detection command, as illustrated in Figure 1 (a). When the correlation structure between these multiple response routes is unknown, it is difficult to fuse these multiple data. One possible way is to select the best route, and the calculated $\sum \log r_{e,i}$ for each route can be used to select the best route.

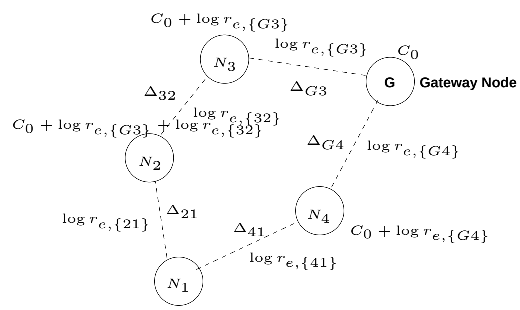

Figure 6: An example of route construction based on the detection performance metric (a dashed line represents that two nodes are communicable neighbors and $C_0 = \log r_{e,0} = \log(\mathrm{SNR}+1)$).

Second, if we accept Assumption 1 for general curves in the interested correlated field, the metric in (26) can be used as a link metric for routing in the conventional sense. Consider the case that a sensor initiates the fusion algorithm based on a local decision and the fusion process proceeds until it reaches the gateway node for global decision. In this case, there may be multiple possible routes to the gateway node, as illustrated in Figure 1 (b), and an optimal fusion route needs to be determined to obtain the best detection performance.

For the routing purpose we utilize the fact that the accumulated sum of $\log r_{e,i}$ is the same for a given route regardless of the direction of the Kalman recursion. That is, for a given route $\mathcal{R}(N_1, \cdots, N_n)$ the sum $\sum_{i=1}^{n} \log r_{e,i}$ is the same when we start the recursion at N_1 finishing at N_n or when we initiate the recursion at N_n ending at N_1. For the route construction we start the Kalman recursion at the gateway node for all paths, and can use routing methods like the Bellman-Ford algorithm. For an example, let us consider the network shown in Figure 6. Suppose that the node N_1 tries to find the best fusion route to the gateway node. Initially, all the nodes set the accumulated cost to the gateway node as $-\infty$ except the gateway node whose cost is $C_0 = \log(\mathrm{SNR}+1)$. In the first iteration, two neighboring nodes N_3 and N_4 of the gateway node calculate the link metric by executing (11 - 15) exchanging the necessary information described in Section A. (That is, N_j requires $P_{j|j-1}$, the accumulated $\sum_{i=1}^{j-1} \log r_{e,i}$, and the location information $\mathbf{x}_{j-1}$ from a communicable neighbor node N_{j-1}.) After the first iteration, two nodes N_3 and N_4 set their accumulated costs which are finite. In the second iteration, N_1 and N_2 update their accumulated cost to finite values bases on paths $\mathcal{R}(\mathrm{G}, N_3, N_2)$ and $\mathcal{R}(\mathrm{G}, N_4, N_1)$, respectively. Now, N_1 has one route to the gateway node. In the third iteration, N_1 will get another route to the gateway node through N_2, and can select the better route.

If we use the approximated link metric (33), exchanging the location information between neighbors is enough for the calculation of the metric. Problems such as loop, oscillation, etc., can be handled similarly to the cases in conventional routing problems.

Third, when each sensor makes multiple observations about a (time-static) phenomenon at its location. The framework described here works without much modification. Suppose that each sensor makes M measurements. Then, the globally optimal statistic can be shown to be based on the observation average of each sensor. Thus, we need only to deal with the same system with one sample per sensor by increasing the SNR by M times.

The overall metric $\sum \log r_{e,i}$ implies that a route with a larger expected innovation at each hop gives better performance. However, to make $R_{e,i}$ larger for a given field correlation we need a longer hop which requires a larger transmission power. Thus, under energy constraint we face the optimization problem of detection-based routing that we are currently working on.

Until now, we have assumed that random fields of interest have the correlation structure described in Assumption 1. Although the assumption is reasonable for almost straight curves in a homogeneous Gauss-Markov random fields, the assumption may be quite unrealistic for general curve shapes, e.g., wiggles. For routes with general geometry in 2-D Gauss-Markov random fields or other reasonable correlated fields, cross-layer approaches combining applications such as detection and estimation directly with the network-layer routing are a challenging open problem.

V. Conclusions

We have considered a cross-layer approach to combining the network-layer routing with the detection at the application layer. We have suggested a framework for the problem and derived a link metric that captures the detection performance. Under the assumption of the Gauss-Markov structure along a given route and high SNR, we have shown that the logarithm of innovation variance is a reasonable choice for a link metric. We have also derived an explicit formula for the proposed metric as a function of the link length. The proposed metric can be easily applied to the conventional routing algorithms. We have also proposed a location-based optimal fusion algorithm and a detection protocol by combining the detection with on-demand routing methods. In the algorithm, the proposed link metric can be used to select an optimal route that provides the best detection performance.

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