ASYMPTOTIC LOCALLY OPTIMAL DETECTOR FOR LARGE-SCALE SENSOR NETWORKS UNDER THE POISSON REGIME

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ABSTRACT

We consider distributed detection with a large number of identical sensors deployed over a region where the phenomenon of interest (POI) has unknown spatially varying strength. Each sensor makes a decision based on its own measurement of the signal at its location and the local decision of each sensor is sent to a fusion center through a multiple access channel. The fusion center decides whether the POI has occurred in the region, under a global size constraint in the Neyman-Pearson formulation. Assuming that the initial distribution of sensors is a homogeneous spatial Poisson process, we show that the Poisson process of 'alarmed' sensors satisfies the locally asymptotic normality (LAN) condition as the number of sensor goes to infinity. We derive a new asymptotically locally most powerful (ALMP) detector jointly over the fusion scheme and the sensor threshold. We also derive the conditions on the spatial signal shape to guarantee the existence of the ALMP detector. We show that the optimal test statistic is a weighted sum of local decisions, the optimal weight function being the shape of the spatial signal, but the exact value of the spatial signal is not required. The optimal threshold for a single sensor is also derived. For the case of independent, identically-distributed (i.i.d.) sensor observation, we show that the counting-based detector is also asymptotic locally optimal.

1. INTRODUCTION

Detection in a large scale microsensor network faces several challenges not encountered in the classical distributed detection problem. First, inexpensive sensors are not reliable; they have low duty cycles and severe energy constraints. The communication link between a sensor and the central unit is specially weak due to a variety of implementation difficulties such as synchronization, fading, and interference from other sensors. The probability that the local decision at a particular sensor can be successfully delivered to the central unit can be very low. Second, POI in a wide geographic area generates spatially varying signals, which makes the observation at each sensor location dependent and not identically distributed. Furthermore, the strength of POI is unknown for many applications such as the detection of biological or chemical agents. Third, the scale of the network makes it more practical to deploy sensors randomly without careful network layout. It is thus not possible to predict whether data from a particular sensor can be retrieved by the the central processing unit, especially when random access protocols are used.

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In this paper, we consider the optimal detection of an unknown spatially varying signal in such large-scale microsensor networks under the Neyman-Pearson context. In particular, we consider the case of identical binary sensors and the asymptotics where the number of sensors goes to infinity within a fixed geographical area. For the asymptotic criterion of distributed detection, the error exponent has been used[11][12]. However, the problem of detecting a signal with unknown strength does not lend itself to the error exponent approach easily due to the uncountable nature of the alternative hypotheses. We consider the asymptotically locally optimal criterion for our problem. Assuming that the initial distribution of sensors forms a homogeneous spatial Poisson process, we convert the problem of global distributed detection to that of spatial Poisson process with different intensity. Using the locally asymptotic normal (LAN) theory from Le Cam, we derive: (1) the sufficient conditions for the spatial signal shape that guarantee the existence of asymptotically locally most power (ALMP) detector, (2) an asymptotic (achievable) upper bound on the power of any detector, and (3) an asymptotically locally optimal rule *jointly* over the fusion scheme and the single sensor threshold. For the special case that the power function of a single sensor is linear, the proposed detector is also asymptotically uniformly most powerful (AUMP).

Related Work

The detection of an unknown signal or a signal with unknown amplitude has been considered by several authors under the composite hypothesis formulation. The locally optimal detector for a centralized scheme is known. Poor and Thomas considered the locally optimal detector for stochastic signals and compared the relative performance of detectors using asymptotic relative efficiency in the centralized detection scheme[8]. For the distributed or decentralized case, Aalo and Viswanathan considered the detection of an unknown signal via multilevel quantization and simple fusion rules[7]. However, no optimality for the fusion rule was considered. Fedele, Izzo, and Paura[9] and Srinivasan[10] considered locally optimal detection of unknown signals. The authors considered a distributed scheme where multiple local detections are combined at the fusion center and the number of observations per each local detector goes to infinity. These assumptions are reasonable for the classical radar problem. However, for large-scale microsensor networks, it is more realistic to assume that each sensor has only a few chances for observations and transmissions due to the limited power and/or duty-cycling, and to consider the asymptotic case where the number of sensors goes to infinity with a limited number of observations per sensor.

2. SYSTEM MODEL

We consider a large-scale sensor network with identical binary sensors deployed over a wide area; we want to decide whether a POI has occurred in the area. Each sensor makes a binary decision, u_i, based on its own observation and the local decisions are collected through a multiple access channel (MAC) at a central unit or fusion center where a global decision is made under a size (PFA) constraint.

We assume that the spatial signal underling the POI is deterministic and denote the strength of the signal by

$$\gamma(\mathbf{x}) = \theta s(\mathbf{x}), \tag{1}$$

where $\mathbf{x}$ denotes the location, $\theta \in \Theta \triangleq [0, \infty)$ is an *unknown* amplitude, and $s(\mathbf{x})$ is a known[1] function which incorporates the information about the spatial variation of the underlying phenomenon.

2.1. Single Sensor

We assume that sensors make their local decisions independently without collaborating with other sensors. Since the exact value of the signal strength is unknown, we design each sensor to solve the following hypothesis testing problem:

$$\begin{array}{ll} H_0: & \gamma(\mathbf{x}) = 0, \\ H_1: & \gamma(\mathbf{x}) > 0, \end{array} \tag{2}$$

with local size constraint of α_0. The hypotheses (2) are equivalently expressed by

$$\begin{array}{ll} H_0: & \theta = 0, \\ H_1: & \theta > 0. \end{array} \tag{3}$$

The local decision of sensor S_i, located at $\mathbf{x}_i$, is denoted by

$$u_i = \begin{cases} 0 & \text{if } H_0 \text{ selected}, \\ 1 & \text{otherwise}. \end{cases} \tag{4}$$

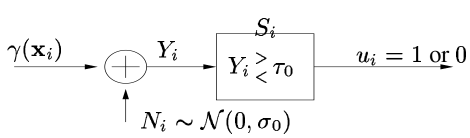

Fig. 1. Sensor located at $\mathbf{x}_i$

One possible sensor observation model is the additive Gaussian noise model shown in Fig. 1, where the sensor input Y_i is given by

$$Y_i = \gamma(\mathbf{x}_i) + N_i, \quad N_i \sim \mathcal{N}(0, \sigma_0), \tag{5}$$

where N_i is the independent sensor noise. In this case, the local decision rule for (3) at each sensor is the UMP detector given by

$$Y_i \underset{H_0}{\overset{H_1}{\gtrless}} \tau_0, \tag{6}$$

[1] We do not assume that $s(\mathbf{x})$ is known beforehand. It can be estimated from the sensor decisions after data collection.

where $\tau_0 = \sigma_0 Q^{-1}(\alpha_0)$. We define the following probability

$$p(\mathbf{x}_i) \triangleq \Pr\{u_i = 1\}. \tag{7}$$

Then, $p(\mathbf{x})$ is a function of the signal strength at $\mathbf{x}$ and is given by

$$p(\mathbf{x}) = g_{\tau_0}(\gamma(\mathbf{x})). \tag{8}$$

For the additive Gaussian noise model, $p(\mathbf{x}) = Q\left(\frac{\tau_0 - \gamma(\mathbf{x})}{\sigma_0}\right)$.

2.2. Parametric Poisson Model

We assume that the initial distribution of sensors over the space is a homogeneous Poisson process with intensity λ_h. Since each sensor decision is independent and based on the signal strength at its location, the local decision making of each sensor can be viewed as a location-dependent thinning procedure of the original sensor distribution with probability $p(\mathbf{x})$ and the distribution of the *alarmed* sensors, i.e., sensors with $u_i = 1$, forms a nonhomogeneous spatial Poisson process. Alarmed sensors encode their decisions and send their packets over an erasure channel with probability of successful transmission p_m; this could include packet losses due to fading as well as collisions. A cross-over channel can also be incorporated since this amounts to changing the local power function, the sensor's $g_{\tau_0}(\cdot)$. Thus, we model this data collection through the MAC as another thinning process.

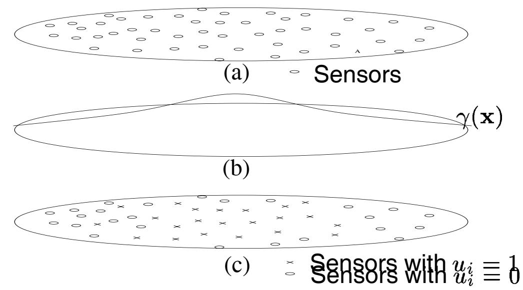

Fig. 2. (a) Initial sensor deployment over area (b) Signal strength of underlying phenomenon (c)Local decisions of sensors

Hence, the distribution of alarmed sensors at the final data collector or fusion center, is a nonhomogeneous Poisson process whose local intensity is given by

$$\lambda(\mathbf{x}) = \lambda_h p_m p(\mathbf{x}) = \lambda_h p_m g_{\tau_0}(\theta s(\mathbf{x})). \tag{9}$$

When the function $g_{\tau_0}(\cdot)$ is linear or θ is in a small neighborhood of $\theta = 0$, the Poisson distribution of alarmed sensors is described by a nonhomogeneous intensity model parameterized by amplitude θ and is given by

$$\lambda(\theta, \mathbf{x}) = \theta f(\mathbf{x}) + \lambda_0, \tag{10}$$

where $f(\mathbf{x}) = \lambda_h p_m g'_{\tau_0}(0) s(\mathbf{x})$, $\lambda_0 = \lambda_h p_m g_{\tau_0}(0)$, and $g'_{\tau_0}(\gamma(\mathbf{x})) = \frac{\partial}{\partial \gamma(\mathbf{x})} g_{\tau_0}(\gamma(\mathbf{x}))$. The Poisson assumption on the initial sensor distribution effectively changes the global detection problem to that of deciding from which intensity model the spatial distribution of alarmed sensors has occurred. Notice that the intensity variation $f(\mathbf{x})$ of alarmed sensors is a scaled version of the spatial signal shape $s(\mathbf{x})$.

3. DETECTION OF SPATIALLY-VARYING SIGNAL

In this section, using the LAN theory[1][2], we derive an asymptotically locally most powerful (ALMP) detector for the problem (3) as the number of sensors goes to infinity in a fixed space and under the Poisson setup and the conditions on $s(\mathbf{x})$ that guarantee the existence of the ALMP detector.

We construct a sequence of statistical models of Poisson processes of alarmed sensors under the Poisson assumption on the initial sensor locations. An asymptotic scenario of infinite number of sensors is easily described by increasing the initial intensity λ_h of sensor deployment.

Model 1 (Fixed area and infinite sensor model) *The intensity of the Poisson process of the initial sensor distribution over the space A with finite area is given by*

$$\lambda_h = n\lambda_{h0}. \tag{11}$$

Then, for each $n \geq 1$, the local intensity of the Poisson process of the alarmed sensors is given, using (10), by

$$\lambda^{(n)}(\theta, \mathbf{x}) = \theta n f(\mathbf{x}) + n\lambda_0, \tag{12}$$

where $f(\mathbf{x}) = \lambda_{h0} p_m g'_{\tau_0}(0) s(\mathbf{x})$ and $\lambda_0 = \lambda_{h0} p_m g_{\tau_0}(0)$. Let $X_A^{(n)}$ denote the realization of the Poisson processes of alarmed sensors on area A. Then, the sequence of probability distributions $\{\mathbf{P}_\theta^{(n)}, \theta \in [0, \infty)\}$ is given by [4]

$$d\mathbf{P}_\theta^{(n)}(X_A^{(n)}) = \exp\left(\sum_{\mathbf{x}_i \in A} \log \lambda^{(n)}(\theta, \mathbf{x}_i) - \int_A \lambda^{(n)}(\theta, \mathbf{x}) d\mathbf{x}\right), \tag{13}$$

where $\mathbf{x}_i$'s are the random points of $X_A^{(n)}$.

Theorem 1 *For Model 1, let the conditions (C.1)-(C.3) be satisfied.*

$$(C.1)\ s(\mathbf{x}) \geq 0, \quad (C.2)\ \sup_{\mathbf{x} \in A} s(\mathbf{x}) < \infty, (C.3)\ \int_A s(\mathbf{x}) d\mathbf{x} > 0.$$

Then, an asymptotic upper bound on the power for any sequence detector ϕ_n with size α, i.e., $\mathbb{E}_{n,0}\phi_n \leq \alpha$, is given by

$$\limsup_{n \to \infty} \sup_{0 < r_n(0)^{-1}\theta \leq M} \left[\mathbb{E}_{n,\theta}\, \phi_n - Q(Q^{-1}(\alpha) - r_n^{-1}(0)\theta)\right] \leq 0. \tag{14}$$

where

$$r_n(0) = n^{-1/2} \lambda_{h0}^{-1/2} p_m^{-1/2} \frac{g_{\tau_0}^{1/2}(0)}{g'_{\tau_0}(0)} \left(\int_A s^2(\mathbf{x}) d\mathbf{x}\right)^{-1/2}. \tag{15}$$

Further, the following sequence of (nonrandomized) detectors is asymptotically locally most powerful with size α for (10, 3):

$$\phi_{n,opt} = \begin{cases} \text{Decide } H_0 & \text{if} \quad \Delta_{n,0} \leq Q^{-1}(\alpha), \\ \text{Decide } H_1 & \text{if} \quad \Delta_{n,0} > Q^{-1}(\alpha), \end{cases} \tag{16}$$

where

$$\Delta_{n,0} = n^{-1/2} \lambda_0^{-1/2} \left(\int_A s^2(\mathbf{x}) d\mathbf{x}\right)^{-1/2} \left(\sum_{i:\, \mathbf{x}_i \in A} s(\mathbf{x}_i) - n\lambda_0 \int_A s(\mathbf{x}) d\mathbf{x}\right). \tag{17}$$

Proof may be found in [13], and is omitted here due to space limitations.

Conditions (C.1)-(C.3) are mild: it suffices for the underlying spatial signal to be non-negative, bounded, and not identically zero over the sensor field. These conditions are general enough to incorporate various spatial variations including step function, linear decay, Gaussian, or exponential decay

$$s(x, y) = e^{-\eta r},\ r = \sqrt{x^2 + y^2}. \tag{18}$$

Notice that the ALMP test statistic $\Delta_{n,0}$ is the weighted sum of alarmed sensors under the Poisson setup. The weight $s(\mathbf{x})$ is the shape of underlying spatial signal $\gamma(\mathbf{x})$. Since $\Delta_{n,0}$ is normalized to have a limit distribution of $\mathcal{N}(0, 1)$, any scaling of $s(\mathbf{x})$ is irrelevant in forming the test statistic. This is in contrast with models and approaches, such as in [5], where the shape and magnitude of the intensity function are required. Theorem 1 describes how to optimally use the knowledge of the signal shape and the locations of the sensors. However, in the next section we will show that the proposed detector is robust with respect to both the shape function $s(\mathbf{x})$ as well as its 'origin'. Note that, as expected, the power of the detector increases monotonically with sensor density, signal strength, and MAC transmission success rate. From (14), we observe that if the signal strength is halved, sensor density must be quadrupled in order to maintain the asymptotic performance. This is consistent with the notion of fusing independent signal decisions. When the POI is uniform over the space ($s(\mathbf{x}) \equiv 1$) or the observation for each sensor is i.i.d., the ALMP test statistic simply becomes the number of alarmed sensors. Hence, for the i.i.d. case, the counting-based rule is ALMP under the Poisson regime.

Another advantage of the Poisson approach is that it facilitates derivation of the optimal threshold for a single sensor. Since the upper bound on the power is achieved asymptotically by the ALMP detector, the optimal local threshold is obtained so as to maximize the upper bound on the global power. The next theorem follows immediately from (14,15).

Theorem 2 *Suppose that the power function $g_{\tau_0}(\cdot)$ for a single sensor is continuous and piecewise differentiable. Under the Poisson regime, the following threshold for a single sensor maximizes the global power for a fixed and sufficiently large number of sensors in the region.*

$$\tau_{opt} = \arg\max_{\tau_0} \frac{g'_{\tau_0}(0)^2}{g_{\tau_0}(0)}. \tag{19}$$

For example, the Gaussian noise model has $g_{\tau_0}(0) = Q(\tau_0/\sigma_0)$, and $g'_{\tau_0}(0) = \frac{1}{\sqrt{2\pi}\sigma_0} \exp(-\frac{1}{2}\left(\frac{\tau_0}{\sigma_0}\right)^2)$. The corresponding local size α_0 for the optimal threshold is 0.2703 which agrees well with the simulation results in Section 4. However, the optimal local threshold doesn't change the increasing rate of power as the number of sensors increases, but the multiplicative coefficient of the rate is affected. Notice that under the assumption of binary decisions and Poisson distributed sensors, the individual sensors need not be very good; a design with a PFA of 0.2703 is optimal! Notice also that the optimal fusion scheme (17) and optimal local threshold (19) do not depend on the unknown strength parameter θ. Hence, the optimal rule is actually the asymptotically uniformly most powerful detector when the Poisson model (10) is true. However, our conversion to the Poisson regime is valid in the local neighborhood of $\theta = 0$ for a typical power function $g_{\tau_0}(\cdot)$.

4. NUMERICAL RESULTS

We considered a two dimensional space A which is circular with radius one. The spatial signal shape we considered is the symmetric exponential in (18) with different decaying rates. The average number of sensors in A was chosen to be 1,000. For the local sensor function, we used the additive Gaussian noise model (5) and the UMP detector with a given local size α_0 described in Section 2.1. For the simulation of power and false alarm probability, 10,000 Monte Carlo runs were executed. For each run, the following procedures were performed. The locations of the sensors were randomly generated according to a homogeneous Poisson process with the given mean intensity. The local threshold was calculated from the local size constraint and set to be the same for all sensors. Zero-mean Gaussian noise with variance one was generated independently for each sensor and added to the signal strength calculated from the location of the sensor and the amplitude parameter to form a sensor observation. Threshold detection was made based on the sum of the signal and noise for the local decision. The global decision was made based on the test statistic $\Delta_{n,0}$ and the number of alarmed sensors for the ALMP detector and the counting-based detector, respectively. The global thresholds for both detectors were determined via the Gaussian limit distribution. Throughout the simulations, the probability of successful data collection from a sensor was set to 0.9. The initial homogeneous density λ_h and the local false alarm probability α_0 were assumed to be known, and the true values were used for the simulation. For the analytic calculation, the linear approximation for the power function $g_{\tau_0}(\cdot)$ was used.

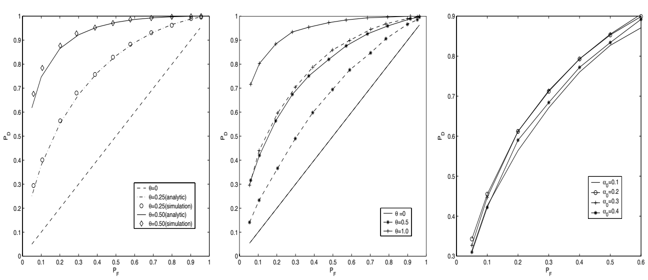

Fig. 3. ROC. Left: analytic vs. simulated($\eta = 3$). Middle: ALMP (solid) vs. counting rule (dashed); $\eta = 6, \alpha_0 = 0.1$. Right: with different local sizes, $\alpha_o = 0.1, 0.2, 0.3, 0.4$; ($\eta = 6$).

Fig. 3 shows the analytic upper bound (14) and simulated powers with respect to the false alarm probability. The power of the ALMP detector almost achieves the upper bound with an average of 1000 sensors in the area. The ALMP detector utilizing the spatial information drastically improves the performance over the counting-based detector. Fig. 3 also shows the ROC with different local sizes. All other parameters were kept the same. The maximum power is attained between $\alpha_0 = 0.2$ and 0.3, which agrees well with the analytic result of $\alpha_o = 0.2703$.

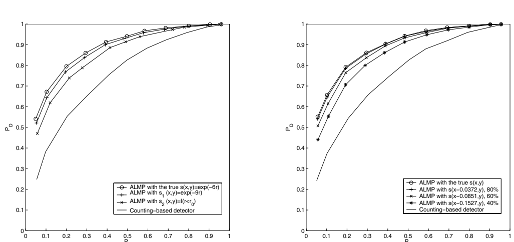

Fig. 4. ROC. Left: mismatched rate. Right: Mismatched center).

Mismatched Case: Fig. 4 shows the ROC of the proposed detector with mismatched signal shapes. The true signal shape of POI was the symmetric exponential with $\eta = 6$. We used two mismatched shapes as the weighting function to construct $\Delta_{n,0}$. First, we considered the symmetric exponential $s_1(x, y)$ with a different decay rate $\eta = 9$. As expected, the proposed detector with the mismatched shape performs worse than the true ALMP detector. Next, we further approximated the signal shape by a step function $s_2(x, y)$ that has 90 % of spatial power compared to the true shape. Even with this rough approximation, the performance degradation was not severe. The other plot shows the ROC with signal shapes with different centers. The true shape was shifted to form mismatched shapes with displacements corresponding to 80,60,40 % of the original peak. The proposed detector also shows robustness to the mismatched center. Hence, a rough estimation of $s(\mathbf{x})$ from the sensor decisions is enough to get the most improvement.

5. CONCLUSIONS

We considered the distributed detection of a spatially varying signal, with unknown strength, using identical sensors that make binary decisions. Assuming Poisson distribution of sensor locations and the availability of location information, we proposed an efficient way to optimize the global decision in the Neyman-Pearson context. We obtained the asymptotically locally optimal detector jointly over the fusion scheme and the sensor threshold. Robustness of the proposed detector was demonstrated via simulations.[2]

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