

# A Least Squares Approach to Joint Beam Design for Interference Alignment in Multiuser Interference Channels

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**Abstract**—The problem of interference alignment for  $K$ -user time-invariant multi-input multi-output (MIMO) interference channels is considered. A sufficient condition for perfect interference alignment is converted to a linear system of equations which is overdetermined for  $K \geq 4$ . It is shown that this linear system is infeasible almost surely. To obtain the approximate solution to this problem, we use a least squares (LS) approach. Various constraints are considered for the LS formulation, including the total signal power, individual user power and individual user beam power constraints. Numerical results are provided to evaluate the proposed design method.

## I. INTRODUCTION

With increasing demand on high data rate wireless communications, emerging wireless communication systems are now being designed with entirely new paradigm. In this paradigm, interference management is one of the key design considerations due to the decreased cell size and introduction of femtocell architecture. As an attractive physical layer technology for interference management, interference alignment is proposed and has gained much attention. It has been shown that for  $K$  user  $M \times M$  MIMO interference channels, one can approximately obtain total degree  $\frac{KM}{2}$  of freedom which is far more than formerly achieved [1], and

$$C(SNR) = \left( \frac{KM}{2} - \epsilon \right) \log(SNR) + o(\log(SNR)), \quad (1)$$

for any  $\epsilon > 0$ . In case of more than three users, however, exact interference alignment scheme is not feasible. To obtain approximate interference aligning beamforming matrices, iterative methods were proposed [2]. The iterative methods obtain beamforming matrices by solving a system of quadratic equations and quadratic optimization. Even though the convergence is guaranteed, the convergence speed depends on the initial condition and stopping rules are heuristic in nature.

To circumvent the problem of iteration, an approach to beam design for interference alignment is proposed under a linear framework [3]. In this method, a system of linear equations was derived based on a sufficient condition for perfect interference alignment, and it was shown that the system is overdetermined almost surely if  $K \geq 4$ . Thus, a least squares approach was used to obtain interference aligning beamforming matrices. The proposed algorithm yields joint beamforming matrices optimal in least squares sense in a single shot. In this paper, we further develop the algorithm

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proposed in [3] by mainly addressing transmission power constraints across users which result in different optimization constraints. From the aspect of interference leakage and effective signal power on the receiver, the proposed algorithm here shows significant improvement over the iterative algorithms in [2].

This paper is organized as follows. The system model is described in Section II. In Section III, the previous work for joint beam design proposed in [3] is described. The improvement of the LS approach is provided in Section IV and V. In Section VI, receiver processing methods are introduced. The performance of the proposed improvement is evaluated in Section VII, followed by conclusion.

## A. Notations

We will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix  $\mathbf{A}$ ,  $\mathbf{A}^H$  indicates the Hermitian transpose of  $\mathbf{A}$ , and  $\text{vec}(\mathbf{A})$  is the column vector consisting of all columns of  $\mathbf{A}$ . For a matrix  $\mathbf{A}$ ,  $\mathbf{A}(i_1 : i_2, j_1 : j_2)$  denotes the submatrix composed of the  $i_1$ -th to  $i_2$ -th rows and  $j_1$ -th to  $j_2$ -th columns of  $\mathbf{A}$ , and  $\mathcal{C}(\mathbf{A})$  represents the column space of  $\mathbf{A}$ , i.e., the linear subspace spanned by the columns of  $\mathbf{A}$ . For matrices  $\mathbf{A}$  and  $\mathbf{B}$ ,  $\mathbf{A} \otimes \mathbf{B}$  denotes the Kronecker product between the two matrices and  $\det(\mathbf{A})$  is a determinant of  $\mathbf{A}$ . For vector  $\mathbf{a}$ , we use  $\|\mathbf{a}\|$  for 2-norm.  $\mathbf{I}_n$  stands for the identity matrix of size  $n \times n$  (the subscript is omitted when unnecessary).  $\langle \cdot, \cdot \rangle$  denotes the inner product operation. The notation  $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$  means that  $\mathbf{x}$  is Gaussian distributed with mean vector  $\boldsymbol{\mu}$  and covariance matrix  $\boldsymbol{\Sigma}$ .

## II. SYSTEM MODEL AND PROBLEM STATEMENT

### A. System model

We consider  $K$ -user  $M \times M$  MIMO interference channels where  $K$  transmitters (each equipped with  $M$  antennas) transmit their signals to  $K$  receivers (each with  $M$  antennas). We assume that the channel state information between all transmitter-receiver pairs is available at all transmitters and receivers. Based on this channel state information, the beamforming matrix at each transmitter maps  $d$  data symbols to  $d$  spatial signatures with length  $M$ , i.e.,  $M$  transmit antennas.

The received signal vector of the  $i$ -th receiver is given by

$$\mathbf{y}_i(t) = \sum_{j=1}^K \frac{P_j}{d_j} \mathbf{H}_{ij}(t) \mathbf{V}_j \mathbf{s}_j(t) + \mathbf{w}_i(t) \quad (2)$$

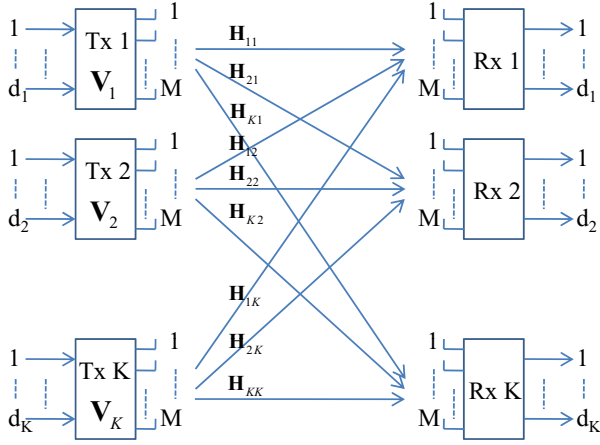


Fig. 1. A  $K$ -user  $M \times M$  MIMO interference channel model.

where  $\mathbf{H}_{ij}(t)$  denotes an  $M \times M$  flat-fading MIMO channel matrix from the  $j$ -th transmitter to the  $i$ -th receiver, an  $M \times d_j$  matrix  $\mathbf{V}_j(t)$  and a  $d_j \times 1$  vector  $\mathbf{s}_j(t)$  represent the beamforming matrix and transmit data symbol vector for the  $j$ -th transmitter, and  $\mathbf{w}_i(t)$  is an  $M \times 1$  complex Gaussian noise vector with distribution  $\mathcal{N}(\mathbf{0}, \mathbf{I})$  at the  $i$ -th receiver. Here,  $d_j$  and  $P_j$  denote the degree of freedom (DoF) or multiplexing gain and the transmit power for the link of the  $j$ -th transmitter, respectively. Therefore, the  $l$ -th column of  $\mathbf{V}_j(t)$  is the spatial signature of the  $l$ -th data symbol stream for  $1 \leq l \leq d_j$  and all  $1 \leq j \leq K$ .

For convenience, we will abbreviate the time index  $t$  assuming block fading in the following sections.

### B. Problem statement

The design goal for interference alignment is to find a set  $\{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_K\}$  of beamforming matrices at the transmitters aligning the interference signal from  $K - 1$  undesired transmitters within a  $(M - d_j)$ -dimensional subspace at each receiver, given the channel state information  $\{\mathbf{H}_{ij}, i, j = 1, \dots, K\}$ . By aligning the interference, we can guarantee an interference-uncorrupted signal space and achieve the multiplexing gain  $d_j$  for each link. In [1], the upper bound of total degree of freedom for  $K$ -user  $M \times M$  MIMO interference channels is shown to be  $\frac{KM}{2}$  and an achievable scheme for  $K = 3$  is introduced. However, the exact alignment is infeasible for  $K \geq 4$ .

In (2), the columns of the effective channel matrix  $\mathbf{H}_{ii}\mathbf{V}_i$  for the desired link span a  $d_i$ -dimensional signal space and those for the interference links  $\mathbf{H}_{i1}\mathbf{V}_1, \dots, \mathbf{H}_{i(i-1)}\mathbf{V}_{(i-1)}, \mathbf{H}_{i(i+1)}\mathbf{V}_{(i+1)}, \dots, \mathbf{H}_{iK}\mathbf{V}_K$  should span a  $(M - d_i)$ -dimensional interference space linearly independent of the signal subspace. If perfect interference alignment is possible, a necessary condition to achieve the upper bound of multiplexing gain  $KM/2$  is that  $d_1 = d_2 = \dots = d_K = \frac{M}{2}$  for given full rank  $M \times M$  channel matrices [3]. Since the upper bound of multiplexing gain per user is  $\frac{M}{2}$ , the condition for perfect interference alignment for  $K$ -user MIMO interference

channels with an even<sup>1</sup> number of antennas is given by

$$\mathcal{C}(\mathbf{H}_{11}\mathbf{V}_2) = \mathcal{C}(\mathbf{H}_{13}\mathbf{V}_3) = \dots = \mathcal{C}(\mathbf{H}_{1K}\mathbf{V}_K), \quad (3)$$

$$\mathcal{C}(\mathbf{H}_{21}\mathbf{V}_1) = \mathcal{C}(\mathbf{H}_{23}\mathbf{V}_3) = \dots = \mathcal{C}(\mathbf{H}_{2K}\mathbf{V}_K), \quad (4)$$

⋮

$$\mathcal{C}(\mathbf{H}_{K1}\mathbf{V}_1) = \mathcal{C}(\mathbf{H}_{K2}\mathbf{V}_2) = \dots = \mathcal{C}(\mathbf{H}_{K(K-1)}\mathbf{V}_{K-1}). \quad (5)$$

Therefore, the objective of interference alignment is to find  $M \times \frac{M}{2}$  beamforming matrices  $\{\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_K\}$  to satisfy the above conditions simultaneously.

### III. PREVIOUS WORK: INTERFERENCE ALIGNMENT BASED ON A LEAST SQUARES APPROACH [3]

In this section, we provide our relevant previous work. An equivalent constraint to (3) - (5) can be formed using orthogonal subspaces, and solving this equivalent condition leads to a set of quadratic equations which is overdetermined for  $K \geq 4$ . Thus, this approach requires iteration to make the system be a overdetermined linear one by fixing one set of variables and apply quadratic optimization. In [3], instead, a sufficient condition for interference alignment (given below) was considered.

$$\mathbf{H}_{12}\mathbf{V}_2 = \mathbf{H}_{13}\mathbf{V}_3 = \dots = \mathbf{H}_{1K}\mathbf{V}_K, \quad (6)$$

$$\mathbf{H}_{21}\mathbf{V}_1 = \mathbf{H}_{23}\mathbf{V}_3 = \dots = \mathbf{H}_{2K}\mathbf{V}_K, \quad (7)$$

⋮

$$\mathbf{H}_{K1}\mathbf{V}_1 = \mathbf{H}_{K2}\mathbf{V}_2 = \dots = \mathbf{H}_{K(K-1)}\mathbf{V}_{K-1}. \quad (8)$$

Note that the  $i$ -th row of the above equations is a condition for interference alignment for the  $i$ -th link, and this is equivalent to a set of  $K(K - 2)$  equalities, i.e.,

$$\mathbf{H}_{12}\mathbf{V}_2 - \mathbf{H}_{1j}\mathbf{V}_j = \mathbf{0}, \quad \forall j \neq 1, 2, \quad (9)$$

$$\mathbf{H}_{i1}\mathbf{V}_1 - \mathbf{H}_{ij}\mathbf{V}_j = \mathbf{0}, \quad \forall j \neq 1, i, \quad \forall i \neq 1. \quad (10)$$

The above equations can be expressed as [3]

$$\tilde{\mathbf{H}}\mathbf{v} = \mathbf{0} \quad (11)$$

where  $\mathbf{v} = [\text{vec}(\mathbf{V}_1)^T, \text{vec}(\mathbf{V}_2)^T, \dots, \text{vec}(\mathbf{V}_K)^T]^T$  and  $\tilde{\mathbf{H}}$  is given by (12). Note that  $\tilde{\mathbf{H}}$  is a  $K(K - 2)\frac{M^2}{2} \times K\frac{M^2}{2}$ . It has been shown that (11) is an overdetermined problem and a nontrivial solution does not exist almost surely when  $K \geq 4$  if the channel coefficients are drawn from the a continuous distribution [3]. Since an exact solution to (11) does not exist for  $K \geq 4$ , an approximate solution that minimizes the 2-norm of total residual interference was considered in [3], i.e.,

$$\hat{\mathbf{v}} = \arg \min_{\|\mathbf{v}\|=1} \|\tilde{\mathbf{H}}\mathbf{v}\|^2. \quad (13)$$

The solution to this is given by the eigenvector corresponding to the smallest eigenvalue of  $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$  [4], [5]. Then, the beamforming matrices can be obtained by proper sectorization and *devec* operation.

<sup>1</sup>In this paper, we only consider interference alignment for even  $M$ . When  $M$  is odd, we can find the same condition for interference alignment by considering two time-slots together as in [1].

$$\tilde{\mathbf{H}} \triangleq \begin{bmatrix} 0 & \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{12} & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{13} & 0 & 0 & \cdots & 0 \\ 0 & \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{12} & 0 & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{14} & \cdots & 0 & \cdots \\ 0 & \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{12} & 0 & 0 & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{15} & 0 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{12} & 0 & 0 & 0 & \cdots & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{1K} \\ \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{21} & 0 & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{23} & 0 & 0 & \cdots & 0 \\ \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{21} & 0 & 0 & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{24} & 0 & \cdots & 0 \\ \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{21} & 0 & 0 & 0 & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{25} & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{21} & 0 & 0 & 0 & 0 & \cdots & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{2K} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{K1} & 0 & 0 & 0 & \cdots & -\mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{K,K-1} & 0 \end{bmatrix} \quad (12)$$

When the fraction of inference power not in the inference space is used for performance measure, the proposed least squares approach shows better performance than the iterative method proposed in [2]. Moreover, the eigenvector associated with the smallest eigenvalue can be easily calculated with an inverse power method [5]. Therefore, the complexity of the least squares method is much lower than that of the iterative scheme.

#### IV. INTERFERENCE ALIGNMENT WITH VARIOUS TRANSMIT POWER CONSTRAINTS

In the previous least squares approach, we employed a global norm constraint  $\|\mathbf{v}\| = 1$  to obtain the solution to minimize overall residual interference easily. In this case, however, the variation of transmit power across users and transmit symbol streams may occur and yield unfair power allocation. In extreme cases, the transmit power of some streams may be reduced to almost zero. To overcome this unfair power allocation of the originally proposed method, we here consider several modified power constraints for minimizing the residual interference power  $\|\tilde{\mathbf{H}}\mathbf{v}\|$ , and the problem reduces to a constrained convex quadratic optimization, which can readily be solved using standard techniques [6].

First, we consider the following optimization problem with individual power constraint.

$$\min \|\tilde{\mathbf{H}}\mathbf{v}\|, \quad (14)$$

subject to

$$\|\mathbf{D}_{11}\mathbf{v}\| = \cdots = \|\mathbf{D}_{1\frac{M}{2}}\mathbf{v}\| = \cdots = \|\mathbf{D}_{K\frac{M}{2}}\mathbf{v}\| = 1, \quad (15)$$

where  $\mathbf{D}_{im}$  is a diagonal matrix (with elements of one and zero) extracting the beamforming vector for the  $m$ -th transmit symbol stream of the  $i$ -th user. Since  $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$  and  $\{\mathbf{D}_{im}\}$  are positive-definite matrices, the optimization problem (14-15) is a convex optimization [6]. To obtain the solution, we define a Lagrangian given by

$$\mathcal{L}(\mathbf{v}, \lambda_1, \dots, \lambda_{K, \frac{M}{2}}) = \mathbf{v}^H \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \mathbf{v} + \sum_{i,m} \lambda_{im} (\mathbf{v}^H \mathbf{D}_{im} \mathbf{v} - 1). \quad (16)$$

By solving  $\frac{\partial \mathcal{L}}{\partial \mathbf{v}} = 0$  and  $\frac{\partial \mathcal{L}}{\partial \lambda_{im}} = 0$  for all  $i$  and  $m$ , we can obtain the beamforming matrices for interference alignment. Therefore, the optimal solution satisfies the following equalities.

$$\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \mathbf{v} + \sum_{i,m} \lambda_{im} \mathbf{D}_{im} \mathbf{v} = 0 \quad (17)$$

and

$$\mathbf{v}^H \mathbf{D}_{im} \mathbf{v} = 1, \quad \forall i, m \quad (18)$$

The solution to (17) and (18) holds the following property which facilitates the calculation of the solution.

*Proposition 1:* If  $(\mathbf{v}, \lambda_{11}, \dots, \lambda_{K, \frac{M}{2}})$  and  $(\mathbf{v}', \lambda'_{11}, \dots, \lambda'_{K, \frac{M}{2}})$  are solutions to (17) and (18),

$$\|\tilde{\mathbf{H}}\mathbf{v}\| \leq \|\tilde{\mathbf{H}}\mathbf{v}'\|$$

when  $\sum_{i,m} \lambda_{im} \geq \sum_{i,m} \lambda'_{im}$ . That is,  $\mathbf{v}$  associated with the maximum  $\sum_{i,m} \lambda_{im}$  is the optimal solution to the optimization problem (14-15).

*Proof:* Because  $(\mathbf{v}, \lambda_{11}, \dots, \lambda_{K, \frac{M}{2}})$  and  $(\mathbf{v}', \lambda'_{11}, \dots, \lambda'_{K, \frac{M}{2}})$  are solutions of (17), we have

$$\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \mathbf{v} + \sum_{i,m} \lambda_{im} \mathbf{D}_{im} \mathbf{v} = 0 \quad (19)$$

$$\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} \mathbf{v}' + \sum_{i,m} \lambda'_{im} \mathbf{D}_{im} \mathbf{v}' = 0 \quad (20)$$

Multiplying (19) by  $\mathbf{v}^H$  and (20) by  $\mathbf{v}'^H$  from the left and subtracting the resulting equations yield

$$\|\tilde{\mathbf{H}}\mathbf{v}\|^2 - \|\tilde{\mathbf{H}}\mathbf{v}'\|^2 + \sum_{i,m} (\lambda_{im} \|\mathbf{D}_{im}\mathbf{v}\|^2 - \lambda'_{im} \|\mathbf{D}_{im}\mathbf{v}'\|^2) = 0. \quad (21)$$

With equalities of (18), we can rewrite (21) as

$$\|\tilde{\mathbf{H}}\mathbf{v}\|^2 - \|\tilde{\mathbf{H}}\mathbf{v}'\|^2 = \sum_{i,m} (\lambda'_{im} - \lambda_{im}). \quad (22)$$

From (17),  $\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} + \sum_{i,m} \lambda_{im} \mathbf{D}_{im}$  should be a singular matrix to have non-zero solution  $\mathbf{v}$ . The modified characteristic polynomial can be defined as

$$\det(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} - \sum_{i,m} \lambda_{im} \mathbf{D}_{im}) \quad (23)$$

where

$$\sum_{i,m} \lambda_{im} \mathbf{D}_{im} = \begin{bmatrix} \lambda_{11} \mathbf{I}_M & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \lambda_{1\frac{M}{2}} \mathbf{I}_M & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \lambda_{21} \mathbf{I}_M & \cdots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \lambda_{K\frac{M}{2}} \mathbf{I}_M \end{bmatrix}$$

First, we find minimum  $\lambda_{im}$ 's make the characteristic polynomial be zero. With these Lagrangian multipliers, we obtain the optimal solution by solving (17) and (18). Summarizing this we have the following algorithm.

*Algorithm: Interference alignment with individual power constraints:*  $M \times M$  MIMO channel matrices  $\{\mathbf{H}_{ij}, i, j = 1, 2, \dots, K\}$  are given.

- 1) Construct  $\tilde{\mathbf{H}}$  given by (12).
- 2) Find a set  $\mathcal{A}$  of  $\lambda_{im}, i = 1, \dots, K, m = 1, \dots, \frac{M}{2}$  such that  $\det(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} - \sum_{i,m} \lambda_{im} \mathbf{D}_{im}) = 0$ .
- 3) Find  $\hat{\lambda}_{im}$ 's in  $\mathcal{A}$  which minimize  $\sum_{i,m} \lambda_{im}$ .
- 4) With  $\hat{\lambda}_{im}$ 's, find  $\hat{\mathbf{v}}$  satisfying (17) and (18).
- 5) Sectorize  $\hat{\mathbf{v}}$  to obtain  $\{\mathbf{V}_i, i = 1, \dots, K\}$ .

## V. INTERFERENCE ALIGNMENT INCORPORATING DESIRED SIGNAL POWER

Until now, we have focused on reducing the residual interference power only. At the  $i$ -th receiver, the interference space is given by  $\mathbf{H}_{i1} \mathbf{V}_1 = \dots = \mathbf{H}_{iK} \mathbf{V}_K$  (if perfectly aligned) and the desired signal space is  $\mathbf{H}_{ii} \mathbf{V}_i$ . The signal space  $\mathcal{C}(\mathbf{H}_{ii} \mathbf{V}_i)$  and interference space  $\mathcal{C}(\mathbf{H}_{ij} \mathbf{V}_j)$  are linearly independent almost surely since the channel coefficients are independently drawn from a continuous distribution. Therefore, we do not need to consider the signal space for pure interference alignment only. However, it is desirable to consider the signal power  $\mathbf{H}_{ii} \mathbf{V}_i$  also due to the noise and remaining fractional interference in the signal space. Thus, our next algorithm incorporates the received signal power  $\|\mathbf{H}_{ii} \mathbf{V}_i\|_F$  in beam design. The modified optimization problem is given by

$$\min \mathbf{v}^H \left( \tilde{\mathbf{H}}^H \tilde{\mathbf{H}} - c \sum_{i=1}^K \Phi_i^H \Phi_i \right) \mathbf{v} \quad (24)$$

subject to

$$\|\mathbf{D}_{11} \mathbf{v}\| = \dots = \|\mathbf{D}_{1\frac{M}{2}} \mathbf{v}\| = \dots = \|\mathbf{D}_{K\frac{M}{2}} \mathbf{v}\| = 1 \quad (25)$$

with a weighting factor  $c > 0$  for the signal power. Here,  $\Phi_i$  is defined as

$$\Phi_i = \text{diag}(\mathbf{0}, \dots, \mathbf{0}, \mathbf{I}_{\frac{M}{2}} \otimes \mathbf{H}_{ii}, \mathbf{0}, \dots, \mathbf{0}) \quad (26)$$

so that  $\mathbf{v}^H \Phi_i^H \Phi_i \mathbf{v} = \|\Phi_i \mathbf{v}\| = \|\mathbf{H}_{ii} \mathbf{V}_i\|_F^2$ . For the interference alignment problem (24 - 25) to be convex, the weighting factor  $c$  is chosen so that  $(\tilde{\mathbf{H}}^H \tilde{\mathbf{H}} - c \sum_{i=1}^K \Phi_i^H \Phi_i)_+$  is a positive definite matrix.

With a Lagrangian of (24 - 25), we have an algorithm for beamformer design incorporating the desired signal power. The algorithm to obtain beamforming matrices is similar with that of interference alignment with individual power constraints in the previous section. Additionally, we can optimize the weighting factor  $c$  depending on weighting between signal power maximization and residual interference minimization.

## VI. RECEIVER PROCESSING

For the receiver processing, we assume that the channel matrices and transmit beamforming matrices are perfectly known at the receivers. Based on this information the  $i$ -th receiver first distinguishes the desired signal and interference subspaces using singular value decomposition:

$$\mathbf{U}_i \Sigma_i \mathbf{V}_i^H = \begin{bmatrix} \mathbf{H}_{i1} \hat{\mathbf{V}}_1, \dots, \mathbf{H}_{i(i-1)} \hat{\mathbf{V}}_{(i-1)}, \mathbf{H}_{i(i+1)} \hat{\mathbf{V}}_{(i+1)}, \dots, \mathbf{H}_{iK} \hat{\mathbf{V}}_K \end{bmatrix}.$$

Based on this SVD the receiver obtain the  $\frac{M}{2}$ -dimensional interference subspace using the columns of  $\mathbf{U}_k$  associated with the  $\frac{M}{2}$  largest singular values, i.e.,

$$\mathbf{S}_{I,i} = \mathbf{U}_i \left( 1 : M, 1 : \frac{M}{2} \right). \quad (27)$$

After the signal and interference subspaces are known, the receiver further applies post-processing to  $\mathbf{y}_i$  based on either the orthogonal projection (i.e., zero-forcing) or whitened matched filtering. At high SNR, the performance gap between the two methods is not significant and thus we consider the orthogonal projection for simplicity here. The final signal output for the orthogonal projection is given by

$$\begin{aligned} \mathbf{z}_i &= \left\langle \mathbf{H}_{ii} \hat{\mathbf{V}}_i, P_{\mathbf{S}_{I,i}^\perp} \mathbf{y}_i \right\rangle \\ &= \hat{\mathbf{V}}_i^H \mathbf{H}_{ii}^H (\mathbf{I} - \mathbf{S}_{I,i} (\mathbf{S}_{I,i}^H \mathbf{S}_{I,i})^{-1} \mathbf{S}_{I,i}^H) \mathbf{y}_i, \end{aligned} \quad (28)$$

and  $\mathbf{z}_i$  contains all information of  $d_i$  transmit symbol streams.

## VII. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed interference alignment methods by simulation, and compare with the iterative interference alignment scheme in [2]. As the performance measure, we use the fractional interference power in the signal space, i.e., the ratio of the power of interference not in the interference subspace to the total interference power, and the fraction of the desired signal power projected on  $\mathbf{S}_{I,i}^\perp$ . The total signal power  $\|\mathbf{H}_{ii} \hat{\mathbf{V}}_i\|^2$  is also evaluated.

We considered  $K \geq 4$  user  $M \times M$  MIMO interference channels of which coefficients are independently generated from a complex Gaussian distribution with zero mean and unit variance. All transmitter send their symbol streams with transmit power  $P_1 = \dots = P_K = 10$ . Additionally, we assume that all  $K$  transmitter send  $d$  streams, i.e.,  $d_1 = \dots = d_K = d$ .

Figures 2 and 3 show the leakage of interference to signal subspace, the projected signal power on  $\mathbf{S}_{I,i}^\perp$ , and signal power  $\|\mathbf{H}_{ii} \hat{\mathbf{V}}_i\|^2$  with different number of transmit symbol streams when  $K = 4$  and  $M = 5$ . Here, the weighting factor  $c$  on signal power is 0.5. In terms of the interference leakage, the LS approaches show better performance than the



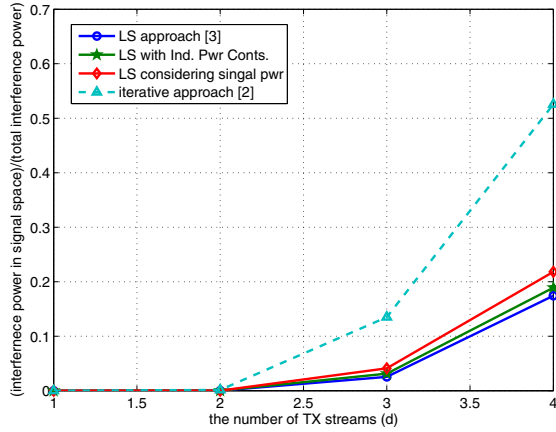


Fig. 2. Fractional interference power in signal space depending on the number of transmit streams ( $K = 4$ ,  $M = 5$ ).

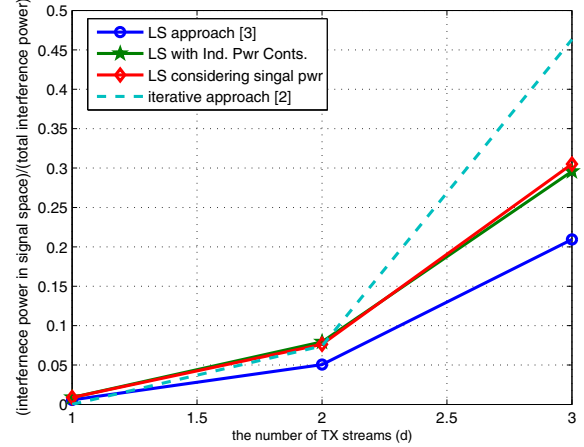


Fig. 4. Fractional interference power in signal space depending on the number of transmit streams ( $K = 5$ ,  $M = 4$ ).

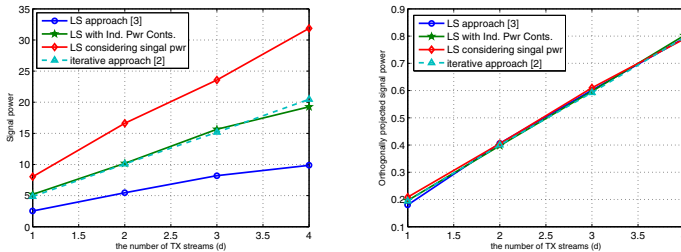


Fig. 3. Signal power (left) and fraction of projected signal power to on  $P_{S,I,i}^\perp$  (right) with different number of transmit streams ( $K = 4$ ,  $M = 5$ ).

iterative method in [2]. Among the LS methods, the simplest LS scheme produces the lowest interference leakage since its constraint on beamforming matrices is looser than those of the others with individual power constraint per transmit stream. The LS approach considering signal power  $\|\mathbf{H}_{ii}\mathbf{V}_i\|$  shows higher interference leakage than the LS method without signal power consideration. Similar performance behavior is shown for different values of  $K$  and  $M$ . As an example, Fig. 4 shows the interference leakage for  $K = 5$  and  $M = 4$ .

To estimate the signal-to-interference and noise ratio (SINR) for the zero-forcing receiver, we also show the signal power and the fractional signal power projected on the orthogonal space of the interference space. The signal power of the simple LS method has lowest signal power because the signal component is not considered in the beamformer design. The LS with power constraint on individual transmit stream and the iterative method which has a unit norm constraint of beamformer have nearly the same signal power. Finally, the LS considering the signal power shows larger signal power.

Since the fraction of signal power contained in the orthogonal space  $\mathbf{S}_{I,i}^\perp$  of the interference space is approximately the same for all algorithms as shown in the right side of Fig. 3, the LS approach incorporating signal power shows better performance than all the others when the orthogonal projection method is used for receiver processing.

## VIII. CONCLUSIONS

In this paper, we have investigated the beam design for interference alignment for  $K$ -user  $M \times M$  MIMO interference channels. Since the exact interference alignment is infeasible for  $K \geq 4$ , we have proposed a design method that approximately aligns the interference based on LS approach. Under our framework, the beam design problem reduces to a constrained convex optimization. We have considered several constraints on beam vectors to incorporate the fairness of power allocation, and have proposed algorithms to solve this convex optimization. Numerical results show that the proposed LS approach outperforms the previously proposed iterative algorithms. The proposed methods have been shown better performance in terms of residual interference. The capacity of proposed algorithms should be checked.

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