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Least Squares Approach to Joint Beam Design for Interference Alignment in Multiuser Multi-Input Multi-Output Interference Channels

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***Abstract*—In this correspondence, the problem of interference alignment for K-user time-invariant multi-input multi-output interference channels is considered. The necessary and sufficient conditions for interference alignment are converted to a system of linear equations that have dummy variables. Based on this linear system, a new algorithm for beam design for interference alignment is proposed by minimizing the overall *interference misalignment*. The proposed algorithm consists of solving a least squares problem iteratively. The convergence of the proposed algorithm is established, and its complexity is analyzed. The performance of the proposed algorithm is also evaluated numerically. It is shown that the proposed algorithm has faster convergence and lower complexity than the previous method with a comparable sum rate performance in the most practical case of two receive antennas.**

***Index Terms*—Interference alignment, interference channels, iterative algorithm, least squares, multi-user MIMO.**

I. Introduction

It has recently been shown that the total number of achievable degrees of freedom (DoF) for K-user $M \times M$ multi-input multi-output (MIMO) time-varying interference channels is $KM/2$ when the *interference alignment* technique is used [1]. With interference alignment, each user can achieve approximately one half of the capacity that can be achieved without interference. Thus, the technique is useful when a wireless network is interference-limited with high signal-to-noise ratio (SNR). For time-invariant MIMO interference channels, it has been shown that $3M/2$ DoF can be achieved for 3-user $M \times M$ MIMO interference channels by using a transmit beamforming approach [1]. However, when $K \geq 4$, the interference alignment with the beamforming approach cannot achieve total $KM/2$ DoF in the $M \times M$ case [2]. Another line of research has focused on the invention of the efficient beam design algorithms for interference alignment in order to implement the potential technique. One such algorithm was presented in [3], which was based on minimizing the interference iteratively from the context of the distributed algorithm. In this correspondence, we propose a new algorithm for

Manuscript received December 17, 2009; accepted May 15, 2010. Date of publication May 24, 2010; date of current version August 11, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Amir Leshem. This work was supported by the Korea Research Foundation Grant funded by the Korean Government (KRF-2008-220-D00079).

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Color versions of one or more of the figures in this correspondence are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/TSP.2010.2051155

interference alignment based on an alternative framework of minimizing the overall *interference misalignment*. Whereas the previous algorithm [3] can be viewed as being based on the idea of orthogonal complement for the equivalence of two linear subspaces from a signal processing perspective, our approach exploits the equivalence of two linear subspaces directly. That is, one basis of a linear subspace is represented by linear combinations of another basis of the same subspace. Exploiting this, we convert the necessary and sufficient conditions for exact interference alignment into a single system of *linear* equations with dummy variables. Based on this new formulation, a new iterative algorithm is constructed for interference alignment by solving the linear system given the value of channel matrices. It is shown that the proposed algorithm has faster convergence and lower complexity than the previous methods in [3] with a comparable sum rate performance when the receivers have two antennas.

A. Related Work and Notation

The interference alignment technique was introduced as a solution to maximize the DoF in K-user interference channels. It was shown that the interference alignment achieved the maximum DoF in K-user time-varying interference channels [1]. Interference alignment on signal scale that used structured coding was also proposed for deterministic channel models [4]. With asymmetric complex signaling, 1.2 DoF can be achieved for $K \geq 3$ for almost all the values of channels [5]. Recently, the achievability of $KM/2$ DoF in time-invariant channels by using a new interference alignment based on the results of number theory was proved [6]–[8]. These schemes were based on the properties of the irrationality of channel coefficients. Other feasibility results for interference alignment are found in [2], [9]. In this correspondence, the algorithmic aspect of the interference alignment based on beamforming is considered, and a new beam design method is proposed. For the algorithmic aspect, Gomadam *et al.* proposed algorithms to minimize the interference and maximize the signal-to-interference-and-noise ratio (SINR) by an iterative approach for distributed implementation [3]. We also introduced initial results of our work based on least squares approach to interference alignment in [10]. Whereas we considered sufficient conditions for interference alignment and proposed non-iterative algorithms in [10], we here propose an iterative least squares algorithm that can realize the necessary and sufficient conditions of interference alignment.

In this correspondence, we will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. $\mathbf{A}^T$ and $\mathbf{A}^H$ indicate the transpose and Hermitian transpose of $\mathbf{A}$, respectively. $\mathrm{vec}(\mathbf{A})$ denotes the column vector that consists of all columns of $\mathbf{A}$, while $\mathrm{rank}(\mathbf{A})$ and $\det(\mathbf{A})$ represent the rank and determinant of $\mathbf{A}$, respectively. $\mathcal{C}(\mathbf{A})$ represents the column space of $\mathbf{A}$, i.e., the linear subspace spanned by the columns of $\mathbf{A}$. Also, $\mathbf{A}^\dagger$ and $\|\mathbf{A}\|_F$ denote the Moore-Penrose pseudo-inverse and Frobenius norm of $\mathbf{A}$, respectively. When a matrix $\mathbf{A}$ is composed of a_{ij}, which is the ith row and jth column element, we express as $\mathbf{A} = [a_{ij}]$. $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product between the two matrices. For a vector $\mathbf{a}$, $\|\mathbf{a}\|$ represents the 2-norm of $\mathbf{a}$. $\mathbf{I}_n$ stands for the identity matrix of size n. The notation $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is complex Gaussian distributed with a mean vector $\boldsymbol{\mu}$ and a covariance matrix $\boldsymbol{\Sigma}$. The set of all users is defined as $\mathcal{K} = \{1, \cdots, K\}$ and $\mathcal{A} \setminus \mathcal{B}$ denotes a set minus operation with two sets $\mathcal{A}$ and $\mathcal{B}$.

II. System Model and Background

We consider a general K-user $M \times N$ MIMO interference channel in which K transmitters (each equipped with N antennas) transmit to K receivers (each equipped with M antennas) simultaneously ($M \leq N$). Due to interference, each user receives the desired signal from its corresponding transmitter and also interference from other undesired transmitters. Thus, the received signal vector at receiver k at symbol time t is given by

$$\mathbf{y}_k[t] = \sum_{l=1}^{K} \mathbf{H}_{kl}[t]\mathbf{x}_l[t] + \mathbf{n}_k[t], \quad t = 1, 2, \cdots, T \tag{1}$$

where $\mathbf{H}_{kl}[t]$ is an $M \times N$ flat MIMO channel matrix from transmitter l to receiver k, $\mathbf{x}_l[t]$ is an $N \times 1$ transmit signal vector with unit power, and $\mathbf{n}_k[t]$ is an $M \times 1$ complex Gaussian noise vector from distribution $\mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$. Furthermore, the transmit signal vector at transmitter l is given by

$$\mathbf{x}_l[t] = \mathbf{V}_l[t]\mathbf{s}_l[t] \tag{2}$$

where $\mathbf{V}_l[t] = [\mathbf{v}_1^{(l)}[t], \cdots, \mathbf{v}_{d_l}^{(l)}[t]]$ is the transmit beamforming matrix of size $N \times d_l$ and $\mathbf{s}_l[t] = [\mathbf{s}_1^{(l)}[t], \cdots, \mathbf{s}_{d_l}^{(l)}[t]]^T$ is the transmit data vector of size $d_l \times 1$. Here, d_l is the number of transmit data streams for the link of the lth transmitter–receiver pair. Note that $\mathbf{v}_m^{(l)}[t]$ is the spatial signature for the mth stream $s_m^{(l)}[t]$ of the lth link $(1 \leq m \leq d_l)$. It is assumed that channels are time-invariant, and this assumption is suitable within a block for the typical block-fading channel model. Thus, the time index t is omitted from here on. We also assume that the channel information is known to the transmitters as well as to the receivers.

A. Background

The basic idea behind interference alignment is to design beamforming matrices $\{\mathbf{V}_l, l \in \mathcal{K}\}$ at the transmitters with given channel information $\{\mathbf{H}_{kl}, k, l \in \mathcal{K}\}$, so that at each receiver, the interference from all the undesired transmitters is aligned or confined within a linear subspace, i.e., interference subspace with a dimension less than that of the observation space at the receiver. In this case of $d_1 = \cdots = d_K = d = M/2$ (that we consider mainly in this correspondence), the condition of perfect interference alignment is given as follows.

Condition 1: Perfect interference alignment in $N \times M$ MIMO channels with $d = M/2$

$$\mathcal{C}(\mathbf{H}_{12}\mathbf{V}_2) = \mathcal{C}(\mathbf{H}_{13}\mathbf{V}_3) = \cdots = \mathcal{C}(\mathbf{H}_{1K}\mathbf{V}_K) \tag{3}$$

$$\mathcal{C}(\mathbf{H}_{21}\mathbf{V}_1) = \mathcal{C}(\mathbf{H}_{23}\mathbf{V}_3) = \cdots = \mathcal{C}(\mathbf{H}_{2K}\mathbf{V}_K) \tag{4}$$

$$\vdots$$

$$\mathcal{C}(\mathbf{H}_{K1}\mathbf{V}_1) = \mathcal{C}(\mathbf{H}_{K2}\mathbf{V}_2) = \cdots = \mathcal{C}(\mathbf{H}_{KK-1}\mathbf{V}_{K-1}). \tag{5}$$

Note that Condition 1 is in the form of subspace equivalence and this requires further elaboration for the condition to be expressed in the form of algebraic equality. Based on orthogonal complement, Condition 1 can be rewritten as follows.

Condition 2: There exist non-zero $\{\mathbf{U}_k : \mathrm{size}(\mathbf{U}_k) = N \times M/2, \mathrm{rank}(\mathbf{U}_k) = M/2, k \in \mathcal{K}\}$ and $\{\mathbf{V}_l : \mathrm{size}(\mathbf{V}_l) = N \times M/2, \mathrm{rank}(\mathbf{V}_l) = M/2, l \in \mathcal{K}\}$ such that [3]

$$\mathbf{U}_k^H\mathbf{H}_{kl}\mathbf{V}_l = \mathbf{0}, \quad k \in \mathcal{K},\ l \in \mathcal{K} \setminus \{k\}. \tag{6}$$

Under this formulation, the problem of interference alignment reduces to finding $\{\mathbf{U}_k, \mathbf{V}_l, k, l \in \mathcal{K}\}$ which satisfy Condition 2 exactly in feasible cases or approximately in infeasible cases. However, the main difficulty in finding an exact or approximate solution to (6) lies in the fact that Condition 2 is a system of *bilinear* equations. To circumvent this difficulty, one can resort to iterative approaches as in [3]. Gomadam *et al.* proposed an iterative interference alignment (IIA) for distributed implementation. In their scheme, $\{\mathbf{V}_l\}$ are first fixed to some initial value in order to make the condition be a system of linear equations in $\{\mathbf{U}_k\}$ (which makes it easy to solve). Then, the solved $\{\mathbf{U}_k\}$ are used to fix the variable $\{\mathbf{U}_k\}$ and to change the condition to be linear in $\{\mathbf{V}_l\}$ for easy solutions to $\{\mathbf{V}_l\}$. These steps are iterated until the solution converges. In addition to this, they considered another approach to maximize the SINR (MAX-SINR) at each receiver.

III. Beam Design for Interference Alignment by Iterative Least Squares

A. Algorithm Construction

While Condition 2 expresses the interference alignment condition effectively by introducing orthogonal space $\mathbf{U}_k$, it converts the problem to solving a system of bilinear equations that requires heavy computational cost. To circumvent this difficulty, orthogonal complements are not used to express the subspace equivalence here, but instead, the structure of Condition 1 is exploited and a linear approach is used to represent the condition. This approach yields a system of linear equations with dummy variables that can be used for algorithm construction. Consider the first equality in the first row of Condition 1:

$$\mathcal{C}\left(\mathbf{H}_{12}\left[\mathbf{v}_1^{(2)},\cdots,\mathbf{v}_d^{(2)}\right]\right)=\mathcal{C}\left(\mathbf{H}_{13}\left[\mathbf{v}_1^{(3)},\cdots,\mathbf{v}_d^{(3)}\right]\right). \quad (7)$$

Note that the equivalence of the column spaces of the two matrices simply implies that a column in one matrix is represented by a linear combination of the columns of the other, and this is a necessary and sufficient condition. Based on this, the subspace equivalence (7) is expressed by the following equation:

$$\mathbf{H}_{12}\mathbf{v}_i^{(2)}=\sum_{j=1}^{d}\alpha_{ij}^{(13)}\mathbf{H}_{13}\mathbf{v}_j^{(3)},\quad i=1,\cdots,d, \quad (8)$$

where $\alpha_{ij}^{(13)}$, $i,j=1,2,\cdots,d$, are the coefficients of linear combination. The key fact here is that the above equation is *linear* in $\mathbf{V}_k$ with dummy variables $\alpha_{ij}^{(13)}$, and $\{\alpha_{ij}^{(13)}\}$ are easy to obtain. (This will be shown shortly.) This facilitates the development of an efficient algorithm. The above multiple equalities in (8) can be written in a matrix-vector form by using Kronecker product as

$$(\mathbf{I}_d\otimes\mathbf{H}_{12})\mathrm{vec}(\mathbf{V}_2)-(\mathbf{A}_{13}\otimes\mathbf{H}_{13})\mathrm{vec}(\mathbf{V}_3)=\mathbf{0} \quad (9)$$

where $\mathbf{A}_{13}=[\alpha_{ij}^{(13)}]$. Now, consider all equalities in Condition 1. Each row in Condition 1 yields $K-2$ independent equalities like (9) with the comparison reference taken as the first element in the first column in Condition 1. Collecting all equalities generated by all K rows, we have the following system of linear equations with dummy variable $\{\mathbf{A}_{kl}\}$:

$$\tilde{\mathbf{H}}\mathbf{v}=\mathbf{0} \quad (10)$$

where $\tilde{\mathbf{H}}$ is defined as (11), shown at the bottom of the page,

$$\mathbf{v}\triangleq\left[vec(\mathbf{V}_1)^T\quad vec(\mathbf{V}_2)^T\quad\cdots\quad vec(\mathbf{V}_K)^T\right]^T, \quad (12)$$

and $\{\mathbf{A}_{kl}\}$ are the matrices composed of linear combination coefficients. Thus, a set of interference-aligning beamforming matrices can be obtained by solving (10) once the channel information $\{\mathbf{H}_{kl}\}$ and the value of $\{\mathbf{A}_{kl}\}$ are given.

On the other hand, we can also write (8) as

$$\mathbf{H}_{12}\mathbf{V}_2=\mathbf{H}_{13}\mathbf{V}_3\mathbf{A}_{13}^T. \quad (13)$$

When $\mathbf{V}_2$ and $\mathbf{V}_3$ are given along with channel information, $\mathbf{A}_{13}$ is directly obtained by using the left inverse and is given in closed form by

$$\mathbf{A}_{13}=\left\{(\mathbf{H}_{13}\mathbf{V}_3)^{\dagger}\mathbf{H}_{12}\mathbf{V}_2\right\}^T \quad (14)$$

since $M\geq d$ and $\mathbf{H}_{13}\mathbf{V}_3$ is a tall matrix. Similarly, all other $\mathbf{A}_{kl}$ are given by

$$\mathbf{A}_{1l}=\left\{(\mathbf{H}_{1l}\hat{\mathbf{V}}_l)^{\dagger}\mathbf{H}_{12}\hat{\mathbf{V}}_2\right\}^T,\quad l\in\{4,\cdots,K\}, \quad (15)$$

$$\mathbf{A}_{kl}=\left\{(\mathbf{H}_{kl}\hat{\mathbf{V}}_l)^{\dagger}\mathbf{H}_{k1}\hat{\mathbf{V}}_1\right\}^T,\quad k\in\mathcal{K}\setminus\{1\},\quad l\in\mathcal{K}\setminus\{1,k\} \quad (16)$$

when $\{\mathbf{V}_l\}$ and $\{\mathbf{H}_{kl}\}$ are given. Note that for $d=1$, in particular, the operation involved in obtaining $\{\mathbf{A}_{kl}\}$ is simply taking inner product. Equations (10) and (14)–(16) are the basic ingredients for our algorithm which will be given next.

B. Basic Algorithm

In this section, we propose a new beam design algorithm for interference alignment based on the results in the previous section. The basic structure of the algorithm is to solve $\{\mathbf{V}_l\}$ and $\{\mathbf{A}_{kl}\}$ iteratively with proper initialization. Note that the existence of the exact solution to (10) depends on the size of $\tilde{\mathbf{H}}$, which is determined by K, M, N, $d_1,\cdots d_K$. When $d_1=\cdots=d_K=M/2$ and $M=N$, for example, the size of $\tilde{\mathbf{H}}$ is $K(K-2)(M^2/2)\times K(M^2/2)$ and the linear system is overdetermined for $K\geq 4$ with an additional constraint on $\mathbf{v}$, as expected. Since an exact solution does not exist in such cases, the least squares approach is applied to (10). The problem can be formulated via least squares as

$$\hat{\mathbf{v}}=\arg\min_{\|\mathbf{v}\|=1}\|\tilde{\mathbf{H}}\mathbf{v}\|. \quad (17)$$

Here, $\|\tilde{\mathbf{H}}\mathbf{v}\|$ can be viewed as the norm of the overall interference misalignment. The solution to (17) is given by the eigenvector associated with the smallest eigenvalue of $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, and the eigenvector associated with the extremal eigenvalues can easily be found by using the power

$$\tilde{\mathbf{H}}\triangleq\begin{bmatrix}\mathbf{0} & \mathbf{I}_d\otimes\mathbf{H}_{12} & -\mathbf{A}_{13}\otimes\mathbf{H}_{13} & \mathbf{0} & \cdots & \mathbf{0}\\ \mathbf{0} & \mathbf{I}_d\otimes\mathbf{H}_{12} & \mathbf{0} & -\mathbf{A}_{14}\otimes\mathbf{H}_{14} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{0} & \mathbf{I}_d\otimes\mathbf{H}_{12} & \mathbf{0} & \cdots & \cdots & -\mathbf{A}_{1K}\otimes\mathbf{H}_{1K}\\ \mathbf{I}_d\otimes\mathbf{H}_{21} & \mathbf{0} & -\mathbf{A}_{23}\otimes\mathbf{H}_{23} & \mathbf{0} & \cdots & \mathbf{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots\\ \mathbf{I}_d\otimes\mathbf{H}_{K1} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{A}_{K(K-1)}\otimes\mathbf{H}_{K(K-1)} & \mathbf{0}\end{bmatrix} \quad (11)$$

method [11]. (In feasible case, $\tilde{\mathbf{H}}$ becomes rank-deficient with proper $\mathbf{A}_{kl}$ and a non-trivial null vector exists.) Thus, the proposed basic algorithm is given as follows:

Algorithm 1: Basic Algorithm: Iterative Least Squares

Step 1. Initialize $\mathbf{A}_{kl}$.
Step 2. Construct $\tilde{\mathbf{H}}$ with $\{\mathbf{H}_{kl}\}$ and $\{\mathbf{A}_{kl}\}$.
Step 3. Obtain $\hat{\mathbf{v}}$ by solving (17) for (10) in least squares sense using the power method.
  3.1 Initialization: $\mathbf{b} = [1, 0, \cdots, 0]$, $\mathbf{R} = c\mathbf{I} - \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$, where a constant c is chosen so that $c\mathbf{I} - \tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ is a positive definite matrix.
  3.2 Repeating the followings steps until it converges: (1) $\mathbf{z} = \mathbf{R}\mathbf{b}$ and (2) $\mathbf{b} = \mathbf{z}/\|\mathbf{z}\|$.
Step 4. Obtain $\{\mathbf{V}_l\}$ by reshaping $\hat{\mathbf{v}}$ from Step 3.
Step 5. Determine $\{\mathbf{A}_{kl}\}$ using $\{\mathbf{V}_l\}$ from Step 4 based on (14)–(16).
Step 6. Iterate Step 2 to 5 until it converges.

C. Modified Algorithms

Here, we provide two variations of the basic algorithm. Algorithm 1 is very fast and efficient, which will be shown shortly in the following section. Due to the global norm constraint $\|\mathbf{v}\| = 1$, however, the variation of transmit power across users and transmit symbol streams may occur and yield unfair power allocation depending on channel realization even if the long-term average power allocation is balanced for i.i.d. channel realization. To overcome this problem we first consider a modified power constraint for minimizing the overall interference misalignment $\|\tilde{\mathbf{H}}\mathbf{v}\|$. The new optimization problem with power constraint P for individual streams is given by

$$\min \quad \|\tilde{\mathbf{H}}\mathbf{v}\| \tag{18}$$

$$\text{subject to} \quad \|\mathbf{D}_{11}\mathbf{v}\| = \cdots = \|\mathbf{D}_{1d_1}\mathbf{v}\| = \cdots = \|\mathbf{D}_{Kd_K}\mathbf{v}\| = P \tag{19}$$

where $\mathbf{D}_{lm}$ is a diagonal matrix (with elements of only one and zero) that extracts the beamforming vector for the mth transmit symbol stream of link l. Since $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$ and $\{\mathbf{D}_{lm}\}$ are positive-definite matrices, the optimization problem (18), (19) can be relaxed as a convex optimization of which Lagrangian is given by

$$\mathcal{L}\left(\mathbf{v}, \lambda_{11}, \cdots, \lambda_{K\frac{M}{2}}\right) = \mathbf{v}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{v} + \sum_{l,m}\lambda_{lm}(\mathbf{v}^H\mathbf{D}_{lm}\mathbf{v} - P). \tag{20}$$

The solution satisfies $\partial\mathcal{L}/\partial\mathbf{v} = 0$ and $\partial\mathcal{L}/\partial\lambda_{lm} = 0$ for all l and m, i.e., $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{v} + \sum_{l,m}\lambda_{lm}\mathbf{D}_{lm}\mathbf{v} = 0$ and $\mathbf{v}^H\mathbf{D}_{lm}\mathbf{v} = P$, since $\mathbf{D}_{lm}^2 = \mathbf{D}_{lm}$. The following property facilitates the computation of the solution to the optimization problem.

Theorem 1: Let $(\mathbf{v}, \lambda_{11}, \cdots, \lambda_{Kd_K})$ and $(\mathbf{v}', \lambda'_{11}, \cdots, \lambda'_{Kd_K})$ be two solutions to $\partial\mathcal{L}/\partial\mathbf{v} = 0$ and $\partial\mathcal{L}/\partial\lambda_{lm} = 0$. Then, $\sum_{l,m}\lambda_{lm} \geq \sum_{l,m}\lambda'_{lm} \Rightarrow \|\tilde{\mathbf{H}}\mathbf{v}\| \leq \|\tilde{\mathbf{H}}\mathbf{v}'\|$. Thus, $\mathbf{v}$ associated with maximum $\sum_{l,m}\lambda_{lm}$ is the solution to the optimization problem (18), (19).

The proof of Theorem 1 can be found in [10]. It is seen from $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{v} + \sum_{l,m}\lambda_{lm}\mathbf{D}_{lm}\mathbf{v} = 0$ that $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \sum_{l,m}\lambda_{lm}\mathbf{D}_{lm}$ is a singular matrix for a non-trivial solution $\mathbf{v}$, i.e.,

$$\det\left(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \sum_{l,m}\lambda_{lm}\mathbf{D}_{lm}\right) = 0, \tag{21}$$

where $\sum_{l,m}\lambda_{lm}\mathbf{D}_{lm} = \mathrm{diag}(\lambda_{11}\mathbf{I}, \cdots, \lambda_{1d_1}\mathbf{I}, \lambda_{21}\mathbf{I}, \cdots, \lambda_{2d_2}\mathbf{I}, \cdots, \lambda_{K1}\mathbf{I}, \cdots \lambda_{Kd_K}\mathbf{I})$. Thus, the solution to (18), (19) is the normalized null space of $(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} - \sum_{l,m}\lambda_{lm}\mathbf{D}_{lm})$ that uses $\{\lambda_{lm}\}$ which satisfies (21) with maximum $\sum_{l,m}\lambda_{lm}$.

Although the optimization problem (18), (19) can be solved by using the algebraic Lagrangian method or the numerical Newton's method [12], we here propose a very efficient algorithm implementing the individual stream power constraint (19). This can be achieved by simply adding a normalization step for each column of $\mathbf{V}_l$ after Step 4 in Algorithm 1. If perfect alignment is feasible, all interference signals are confined in a subspace that is linearly independent of the signal subspace. Therefore, the scaling of beamforming matrices does not affect the interference subspace. Thus, the normalization step can freely be added. With this additional step to Algorithm 1, the overall interference misalignment is minimized while the individual stream power constraint is satisfied.

Algorithm 2: With Individual Power Constraint)

Step 4′. Obtain $\{\mathbf{V}_l\}$ by reshaping and normalizing $\hat{\mathbf{v}}$ from Step 3.
  4′.1 Obtain $\{\mathbf{V}_l = [\mathbf{v}_1^{(l)}, \cdots, \mathbf{v}_d^{(l)}]\}$ by reshaping $\hat{\mathbf{v}}$.
  4′.2 Normalize $\mathbf{v}_m^{(l)}$ such that $\|\mathbf{v}_m^{(l)}\| = 1, m = 1, \cdots d, l = 1, \cdots, K$.

All other steps of Algorithm 2, except for Step 4, are the same as those of Algorithm 1.

In Algorithms 1 and 2, we focused on reducing the overall interference misalignment $\|\tilde{\mathbf{H}}\mathbf{v}\|$ only. At receiver k, the interference subspace is given by $\mathcal{C}(\mathbf{H}_{k1}\mathbf{V}_1) = \cdots = \mathcal{C}(\mathbf{H}_{k,k-1}\mathbf{V}_{k-1}) = \mathcal{C}(\mathbf{H}_{k,k+1}\mathbf{V}_{k+1}) = \cdots = \mathcal{C}(\mathbf{H}_{kK}\mathbf{V}_K)$ (if perfectly aligned) and the desired signal subspace is $\mathcal{C}(\mathbf{H}_{kk}\mathbf{V}_k)$. The signal subspace $\mathcal{C}(\mathbf{H}_{kk}\mathbf{V}_k)$ and interference subspace $\mathcal{C}(\mathbf{H}_{kl}\mathbf{V}_l)$ are linearly independent almost surely for randomly realized channel $\{\mathbf{H}_{kl}\}$. Therefore, it is not necessary to consider the signal subspace for interference alignment purpose only. However, it is desirable to take the power of signal $\mathbf{H}_{kk}\mathbf{V}_k$ also into consideration due to the noise and the remaining fractional interference in the signal subspace. Thus, our next algorithm incorporates the received signal power $\|\mathbf{H}_{kk}\mathbf{V}_k\|_F$ into the beam design, and the modified optimization problem is given by

$$\min \quad \mathbf{v}^H(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} - \gamma\mathbf{\Phi}^H\mathbf{\Phi})_+\mathbf{v} \tag{22}$$

$$\text{subject to} \quad \|\mathbf{D}_{11}\mathbf{v}\| = \cdots = \|\mathbf{D}_{1d_1}\mathbf{v}\| = \cdots = \|\mathbf{D}_{Kd_K}\mathbf{v}\| = P \tag{23}$$

with a weighting factor $\gamma \geq 0$ for the signal power. Here, $\mathbf{\Phi}$ is defined as $\mathbf{\Phi} = \mathrm{diag}(\mathbf{I}_{d_1} \otimes \mathbf{H}_{11}, \cdots, \mathbf{I}_{d_K} \otimes \mathbf{H}_{KK})$, so that $\mathbf{v}^H\mathbf{\Phi}^H\mathbf{\Phi}\mathbf{v} = \|\mathbf{\Phi}\mathbf{v}\|^2 = \sum_{k=1}^K \|\mathbf{H}_{kk}\mathbf{V}_k\|_F^2$. The weighting factor γ is chosen so that $(\tilde{\mathbf{H}}^H\tilde{\mathbf{H}} - \gamma\mathbf{\Phi}^H\mathbf{\Phi})_+$ is a positive definite matrix and the optimization (22), (23) can be relaxed as a convex problem. The algorithm can easily be obtained by simply modifying Algorithm 2, and is given by the following.

Algorithm 3: Incorporating Signal Power

Step 3″. Perform Step 3 in Algorithm 2 with $\mathbf{R} = c'\mathbf{I} - \tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \gamma\mathbf{\Phi}^H\mathbf{\Phi}$, where a constant c' is chosen so that $c'\mathbf{I} - \tilde{\mathbf{H}}^H\tilde{\mathbf{H}} + \gamma\mathbf{\Phi}^H\mathbf{\Phi}$ is a positive definite matrix.

Algorithm 3 is similar to Algorithm 2, except that it has a modified matrix $\mathbf{R}$ that incorporates the signal power. Algorithm 3 can be interpreted as the optimization problem of minimizing the interference misalignment $\mathbf{v}^H\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}\mathbf{v}$ satisfying a certain level of total signal power, i.e., $\mathbf{v}^H\mathbf{\Phi}^H\mathbf{\Phi}\mathbf{v} \geq \xi$. Here, γ is the Lagrange multiplier for the

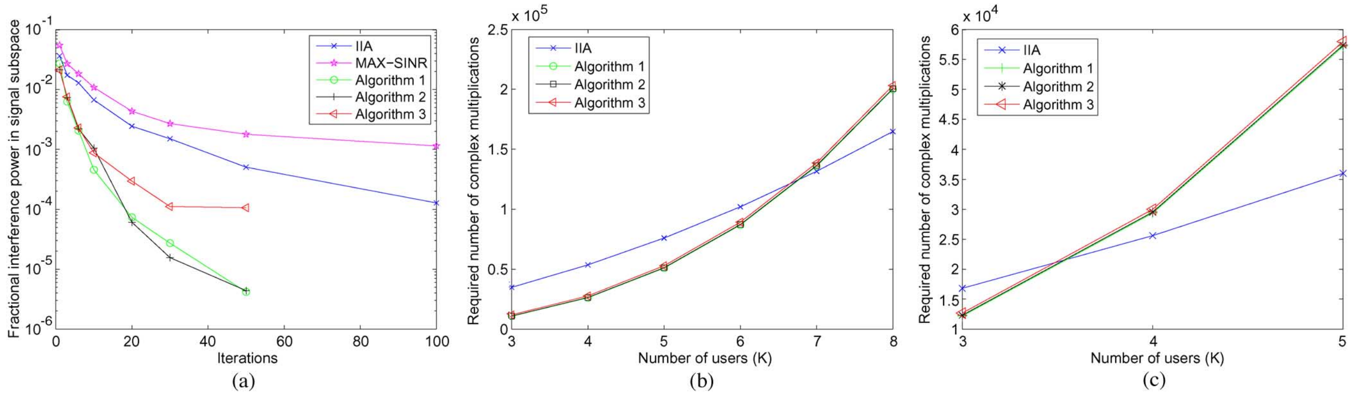

Fig. 1. (a) Fraction of interference leakage into signal space ($M = N = 2, K = 3, d = 1$), (b) total number of complex multiplications ($M = 2, N = 4$) and (c) total number of complex multiplications ($M = N = 2$)

dual optimization problem. We can also select the weighting factor γ judiciously depending on the compromise between the signal power maximization and the interference misalignment minimization. Algorithm 3 is closely related to the MAX-SINR algorithm in [3] because both methods consider the desired signal, as well as the interference. The MAX-SINR algorithm performs a minimum mean square error (MMSE) combining operation with given transmit beam matrices at each receiver, but we make the total desired signal and interference balanced with weight γ. Throughout this correspondence, we set a heuristic value $\gamma = 1/\text{SNR}$ to balance noise power and interference power where SNR is defined as the average of all desired link SNR values.

D. Discussion

First, the proposed algorithms are developed for centralized operations. That is, the proposed algorithms require that all transmitters know all channel information and all transmit beams are designed simultaneously. Such a scenario is reasonable for downlinks with basestation cooperation. Second, one drawback of the currently proposed algorithms is that it is difficult to handle the interference from other data streams of the same transmitter in the linear formulation presented in Section III-A. Thus, the proposed algorithms realize fast convergence and less complexity (which will be shown shortly) with the cost of non-orthogonality among the multiple streams of the same user. (The sum-rate performance in the 4 × 4 MIMO case is shown in Fig. 2(c).) The correction to this problem should be investigated further. However, this is not a problem in practical downlink systems in which receivers, i.e., mobile stations, mostly have two antennas ($M = 2$).

IV. Convergence and Complexity

In this section, we first show the convergence of the proposed iterative least squares algorithms for the beam design of the interference alignment presented in the previous section.

Theorem 2 (Convergence): The iterative least squares algorithms based on (10) and (14)–(16) in Section III converge.

Proof: Let $\tilde{\mathbf{H}}(\{\mathbf{A}_{kl}[n-1]\})$ denote the matrix $\tilde{\mathbf{H}}$ defined as (11) based on $\{\mathbf{A}_{kl}[n-1]\}$. Then, $\hat{\mathbf{v}}[n]$ at the nth iteration is obtained by $\hat{\mathbf{v}}[n] = \arg\min_{\mathbf{v}} \|\tilde{\mathbf{H}}(\{\mathbf{A}_{kl}[n-1]\})\mathbf{v}\|$ under the constraint on $\mathbf{v}$ corresponding to each algorithm. Then,

$$I[n] \triangleq \left\|\tilde{\mathbf{H}}\left(\{\mathbf{A}_{kl}[n-1]\}\right)\hat{\mathbf{v}}[n]\right\| \geq \left\|\tilde{\mathbf{H}}\left(\{\mathbf{A}_{kl}[n]\}\right)\hat{\mathbf{v}}[n]\right\|. \quad (24)$$

The inequality (24) is because $\{\mathbf{A}_{kl}[n]\}$ itself is the least squares solution to minimize $\Delta_{kl}[n] \triangleq \|\mathbf{H}_{kl'}\hat{\mathbf{V}}_{l'}[n] - \mathbf{H}_{kl}\hat{\mathbf{V}}_{l}[n]\mathbf{A}_{kl}\|_F = \|(\mathbf{I} \otimes \mathbf{H}_{kl'})\text{vec}(\hat{\mathbf{V}}_{l'}[n]) - (\mathbf{A}_{kl} \otimes \mathbf{H}_{kl})\text{vec}(\hat{\mathbf{V}}_{l}[n])\|$ for given $\hat{\mathbf{v}}[n] = \text{vec}([\hat{\mathbf{V}}_1[n], \cdots, \hat{\mathbf{V}}_K[n]])$, and $\|\tilde{\mathbf{H}}\hat{\mathbf{v}}[n]\|^2 = \sum_{k,l} \Delta_{kl}^2[n]$. Since $\hat{\mathbf{v}}[n+1] = \arg\min_{\mathbf{v}} \|\tilde{\mathbf{H}}(\{\mathbf{A}_{kl}[n]\})\mathbf{v}\|$, we have

$$I[n+1] = \left\|\tilde{\mathbf{H}}\left(\{\mathbf{A}_{kl}[n]\}\right)\hat{\mathbf{v}}[n+1]\right\| \leq \left\|\tilde{\mathbf{H}}\left(\{\mathbf{A}_{kl}[n]\}\right)\hat{\mathbf{v}}[n]\right\|. \quad (25)$$

Finally, combining (24) and (25) yields $I[n+1] \leq I[n]$. Hence, the norm of interference misalignment decreases monotonically as the number iteration increases, and the algorithms converge by the monotone convergence theorem since the norm of interference misalignment is lower bounded by zero. ■

Theorem 2 shows the convergence of the proposed iterative least squares algorithms via the convergence of the norm of interference misalignment, i.e., $\|\tilde{\mathbf{H}}\mathbf{v}\|$. Now, we compare the convergence speed of the proposed algorithms and the previous methods in [3]. In both cases, the total interference leakage into the signal subspace, instead of interference misalignment, is used for performance measure. Fig. 1(a) shows the fraction of interference in the signal subspace with respect to the iteration number with 20 dB SNR. The interference subspace is defined as the subspace that is spanned by the $M-d$ dominant eigenvectors of the $M \times M$ interference covariance matrix, and the signal subspace is defined as the subspace spanned by the remaining eigenvectors, i.e., d smallest eigenvectors. The fraction of interference in the signal subspace is defined as the sum of d smallest eigenvalues of an interference covariance matrix divided by total interference power [3]. It is seen that the proposed algorithms converge much faster than the previous methods. Note also that the leakage level of Algorithm 3 is higher than that of Algorithm 1 because of the additional optimization goal of increasing the signal power. However, it will be shown later that this is not detrimental to the sum rate performance.

A. Complexity

Now, we analyze the computational complexity of the proposed algorithms, and compare them with that of the previous method. We consider the number of complex multiplications as complexity criterion. Here, only the case of $M = 2$ and $d = 1$ is considered since this practical case is the main focus of the application of the proposed algorithms, as discussion in Section III-D, and the previous IIA method in [3] is considered only since the MAX-SINR algorithm that yields a larger sum rate in low SNR requires more complexity. The number of complex multiplications of the previous IIA method and Algorithm 1 is summarized in Table I. The total number of multiplications for each algorithm is the product of the number of iterations and the sum of the numbers in each table. The complexity of Algorithms 2 and 3 is also similarly given as that of Algorithm 1 with slight modification of the normalization step. The numbers of iterations are determined to be 50

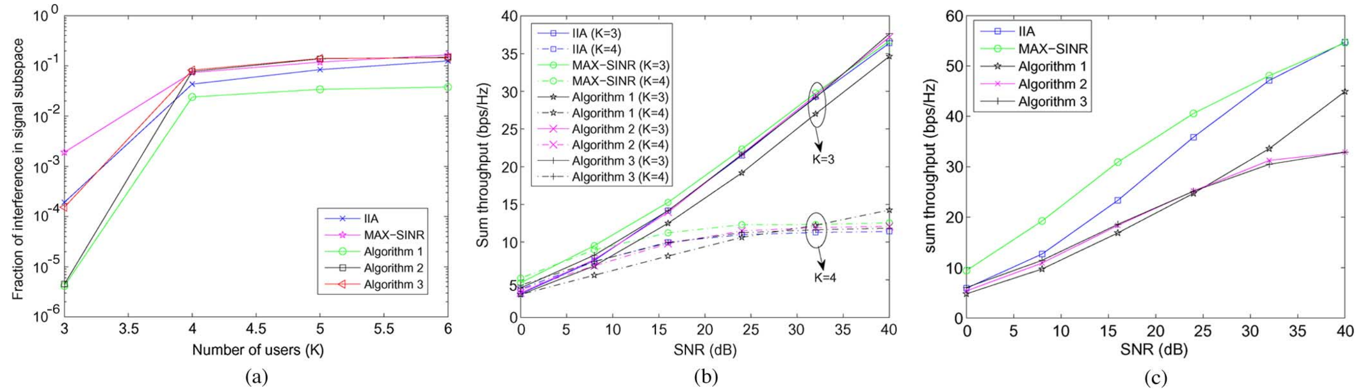

Fig. 2. (a) Fraction of interference leakage into signal space ($M = N = 2, K = 3, d = 1$), (b) sum rate ($M = N = 2, K = 3, 4, d = 1$) and (c) sum rate ($M = N = 4, K = 3, d = 2$).

TABLE I
NUMBER OF COMPLEX MULTIPLICATIONS FOR IIA AND ALGORITHM 1 ($\{\mathbf{Q}_k\}$ AND $\{\bar{\mathbf{Q}}_k\}$ ARE INTERFERENCE COVARIANCE MATRICES OF FORWARD AND REVERSE LINKS)

IIA		Algorithm 1	
Step	Complex multiplications	Step	Complex multiplications
Compute $\{\mathbf{Q}_k\}$ with $\{\mathbf{U}_k\}$	$K(K-1)(MN+M^2)$	2	$K(K-2)MN$
Find the smallest eigenvector	$KL(M^2+2M)$	3 - Compute $\tilde{\mathbf{H}}^H\tilde{\mathbf{H}}$	$(K-2)(5K-4)NM^2$
Compute $\{\bar{\mathbf{Q}}_k\}$ with $\{\mathbf{V}_k\}$	$K(K-1)(MN+N^2)$	3 - Power method	$L(K^2N^2+2KN)$
Find the smallest eigenvector	$KL(N^2+2N)$	5	$K(K-1)MN+K(K-2)(2M+1)$

and 8 for the IIA and Algorithm 1, respectively, based on the results in Fig. 1(a) so that each algorithm terminates with the same interference leakage of 10^{-3}. The number of iterations for the power method used in the proposed algorithms was set to $L = 5$ since the mean square error of eigenvector becomes lower than -40 dB with this number of iterations. Fig. 1(b) and (c) shows the total number of complex multiplications for each algorithm. It is seen in both cases that the proposed algorithms have less complexity than the IIA algorithm for the values of K for which exact interference alignment is feasible. For the values beyond this point the proposed algorithms show a little larger complexity than the previous method.

V. NUMERICAL RESULTS

In this section, we evaluate the performance of the proposed algorithms numerically. Fig. 2 shows the interference leakage into the signal subspace for the previous algorithms and the proposed three algorithms after convergence for a case of $M = 2$, $N = 2$ and $d = 1$ with 20 dB SNR. As expected, Algorithm 1 yields the least leakage into the signal subspace because of the largest freedom in choosing $\mathbf{v}$, whereas the leakage is large with MAX-SINR and Algorithm 3 because of the additional optimization goal of the signal power increase. The leakage performance of Algorithm 2 is in-between. It is also seen that the leakage does not increase further after the critical value of K. Since the final performance measure is sum rate, we evaluated the sum rate for all five algorithms. The exact alignment is feasible for $K = 3$ and is not for $K = 4$, which is clearly shown in the figure. Here, MAX-SINR, IIA, Algorithm 2 and 3 show almost the same performance in the high SNR region. For $K = 3$, all five algorithms shows the same slope of sum rate, i.e., DoF of all algorithms are the same. The sum rate performance in the case of $N = M = 4$ and $d = 2$ is shown in Fig. 2(c). As mentioned in Section III-D, the proposed algorithms show the performance degradation since the interference from data streams of the same user is not properly handled in the currently proposed algorithms. (More numerical results can be found in [13].)

VI. CONCLUSION

We have considered the beam design for interference alignment for K-user time-invariant MIMO interference channels. We have provided an alternative framework for interference aligning beam design by converting the necessary and sufficient conditions for interference alignment into a system of linear equations with dummy variables. Based on this new formulation, we have proposed new algorithms for interference alignment that can solve a least squares problem iteratively. In the practical case of two receive antennas, the proposed algorithm shows fast convergence and less complexity with comparable sum rate performance.

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Joint Receive-Transmit Beamforming for Multi-Antenna Relaying Schemes

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***Abstract*—In this correspondence, we study the problem of joint receive and transmit beamforming for a wireless network consisting of a transmitter, a receiver, and a relay node. The relay node is equipped with multiple antennas while the transmitter and the receiver each uses only one antenna. Our communication scheme consists of two phases: first the transmitter sends the information symbols to the relay. In the second phase, the relay re-transmits a linearly transformed version of the vector of the signals received at its multiple antennas. We introduce the novel concept of general rank beamforming which can be applied to our communication scheme. In our general rank beamforming approach, the relay multiplies the vector of its received signals by a general-rank complex matrix and re-transmits each entry of the output vector on the corresponding antenna. Through maximizing the signal-to-noise ratio (SNR), we obtain a closed-form solution to the general rank beamforming problem. We also prove that for the case of statistically independent transmitter-relay (TR) and relay-receiver (RR) channels, the general rank beamforming approach results in a rank-one solution for the beamforming matrix regardless of the rank of the channel correlation matrices. Simulation results show that when applied to the case of statistically dependent TR and RR channels, our general rank beamforming technique outperforms the separable receive and transmit beamforming method by a significant margin.**

***Index Terms*—Cooperative communications, receive beamforming, relay-assisted communications, transmit beamforming.**

I. Introduction

Recently, multi-user cooperation diversity has been the focus of research in the area of wireless communications [1]–[3]. Cooperative communications exploits the spatial diversity of different users in the network to alleviate the need for using multiple antennas for each user [4]. In multi-user cooperative communications, different users can relay messages originating from a certain user towards the destination thereby providing multiple paths from the source to the destination.

Different aspects of employing relay networks (such as capacity enhancement, diversity gain, and performance improvement) have been studied in the literature [5]–[7]. Also, numerous cooperation schemes, including distributed space-time coding and decentralized beamforming, have been presented in the literature [8]–[19].

The distributed beamforming schemes presented in [16]–[18] assume that the transmitter, the receiver, and the relay nodes all use a single antenna. As a result, these schemes do not benefit from spatial processing at the nodes. Using multiple antennas at the relays allows (locally-implemented) beamforming techniques to be used to improve the quality of the transmitted signal, thereby enhancing the quality of the signal received at the receiver. Several published reports have considered relaying schemes with multi-antenna relay nodes. In [20], a non-regenerative multiple-antenna relaying strategy is developed through the maximization of the capacity between the source and the destination. The resulting beamforming matrix turns out to be full rank. The relaying scheme of [20] has been studied in [21], where an SNR maximization approach is used to obtain the beamforming matrix. Assuming that the receiver and the relay nodes have the perfect knowledge of all their instantaneous receive channel state information, the authors of [21] show that the beamforming matrix is rank one.

In this correspondence, we consider a network consisting of a transmitter, a receiver, and a relaying node (Fig. 1). The transmitter and the receiver have only one antenna while the relay node is equipped with multiple antennas. We herein assume that *only the second-order statistics of the source-relay and relay-destination channel vectors are available*.

As the multiple antennas are located at the same location, their signals can be fully used to establish a connection between the transmitter and the receiver. To do so, one straightforward approach is to use a separable receive-transmit beamforming scheme. That is, one can use a receive beamforming at the relay to linearly estimate the transmitted signals by maximizing the SNR at the output of the receive beamformer. Then, given the output of the receive beamformer, a transmit beamformer can be designed to re-transmit the so-obtained signal estimate such that the receiver's SNR is maximized subject to a constraint on the relay transmit power. Alternatively, one can design the receive and transmit beamformers jointly through the maximization of the receiver's SNR subject to a constraint on the total relay transmit power. It is easy to prove that the separable receive-transmit beamforming approach is equivalent to the joint receive-transmit beamforming technique.

We herein introduce the novel approach of *general rank beamforming*, which was originally proposed in [22] and used in [23]. In this approach, the relay multiplies the vector of its received signals by a *beamforming matrix* rather than a beamforming weight vector (as in the aforementioned receive-transmit beamforming techniques). *We show that using our general rank beamforming matrix, the problem of maximization of the receiver's SNR, subject to constrained relay transmit power, leads to a closed-form solution, even if the correlation matrices of the source-relay and relay-destination channel vectors are full rank*. We further prove that for the case of independent channels, our general rank beamforming technique is exactly equivalent to the separable receive-transmit beamforming approach, and therefore, they both achieve the same maximum SNR. This is rather surprising as one expects that due to additional degrees of freedom (DoF) offered by the general rank beamformer, it should outperform the separable receive-transmit beamformer. Our proof shows otherwise.

Manuscript received September 03, 2009; accepted April 27, 2010. Date of publication June 01, 2010; date of current version August 11, 2010. The associate editor coordinating the review of this manuscript and approving it for publication was Prof. Hongbin Li. This work was supported by Natural Sciences and Engineering Research Council (NSERC) of Canada through Discovery Grant program. This work was presented in part at the IEEE International Conference on Communications, Dresden, Germany, 2009.

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Digital Object Identifier 10.1109/TSP.2010.2051431