ADAPTIVE BEAM TRACKING FOR INTERFERENCE ALIGNMENT FOR MULTIUSER TIME-VARYING MIMO INTERFERENCE CHANNELS

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ABSTRACT

The problem of interference alignment in time-varying MIMO interference channels is considered. To reduce complexity, an adaptive algorithm for beam vector design is proposed based on our previous work of least squares approach to beam design for interference alignment and matrix perturbation theory. The proposed algorithm calculates interference-aligning beam vectors by additive update of the previous value and reduces complexity significantly. Numerical results are provided to validate the proposed algorithm. It is shown that the proposed adaptive algorithm yields almost the same performance as a non-adaptive method that calculates interference-aligning beam vectors at every time step.

Index Terms- Interference alignment, least squares, time-varying channels, adaptive algorithm, matrix perturbation theory

1. INTRODUCTION

Interference alignment is one of the most attractive interference management methods requiring global channel information but not sharing data in multiuser multiple-input multiple-output (MIMO) interference channels. When data sharing is possible, it is known that the dirty paper coding (DPC) is the optimal scheme in terms of achievable rate [1]. Otherwise, the interference alignment achieves the maximum degree of freedom (DoF) of $\frac{KM}{2}$ [2]. The interference alignment was first introduced in the X-channels [3, 4] and applied to K-user interference channels [2]. An iterative algorithm to find interference-aligning beamforming vectors for MIMO interference channels was proposed in [5] based on maximum signal-to-noise and interference ratio. A different interference alignment method minimizing overall interference mis-alignment based on least squares approach was proposed in [6, 7].

While most results regarding interference alignment have been focused on achievability and beam design with perfect channel knowledge, not much attention has been paid to the problem of interference alignment in time-varying MIMO channels and the investigation of the impact of imperfect channel knowledge on interference alignment [8]. In this paper, we consider the problem of adaptive beam design for interference alignment in time-varying MIMO channels which is the case in typical wireless systems. The benefit of adaptive or recursive realization is profound in many important signal processing algorithms such as Kalman filter, recursive least squares (RLS), least mean-square (LMS) algorithm, etc., and this is also true for interference alignment to reduce complexity and for the idea to be used in practice. Our approach to adaptive realization is based on our previous work of least squares (LS) approach to interference alignment [6, 7]. In this formulation, the condition for interference alignment is expressed by a system of linear equations with dummy variables, and this enables us to realize an adaptive algorithm for interference alignment using the matrix perturbation theory. (The detail will be explained shortly.) The proposed algorithm calculates beam vectors at time $n + p$ based on the value at time $n - 1$ for $p = 0, 1, \cdots, L - 1$ as the pure Kalman prediction steps, and reduces the required complexity significantly.

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1.1. Notations

We will make use of standard notational conventions. Vectors and matrices are written in boldface with matrices in capitals. All vectors are column vectors. For a matrix $\mathbf{A}$, $\mathbf{A}^T$ and $\mathbf{A}^H$ indicate the transpose and Hermitian transpose of $\mathbf{A}$, respectively, and $vec(\mathbf{A})$ is the column vector consisting of all the columns of $\mathbf{A}$. For a matrix $\mathbf{A}$, $\mathcal{C}(\mathbf{A})$ represents the column space of $\mathbf{A}$, i.e., the linear subspace spanned by the columns of $\mathbf{A}$. $\mathbf{A} = [a_{ij}]$ means that $\mathbf{A}$ is a matrix composed of a_{ij} with the i-th row and j-th column element. $\mathbf{A}^{i,j}$ denotes the (i, j) element of $\mathbf{A}$. For matrices $\mathbf{A}$ and $\mathbf{B}$, $\mathbf{A} \otimes \mathbf{B}$ denotes the Kronecker product between the two matrices. $\mathbf{A}^\dagger$ denotes the pseudo inverse of matrix $\mathbf{A}$. We use $\|\mathbf{a}\|$ for 2-norm of vector $\mathbf{a}$, $\|\mathbf{A}\|_F$ and $||\mathbf{A}||_2$ denote the Frobenius norm and 2-norm of a matrix $\mathbf{A}$, respectively. $\mathbf{I}_n$ stands for the identity matrix of size $n \times n$ (the subscript is omitted when unnecessary). The notation $\mathbf{x} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ means that $\mathbf{x}$ is complex Gaussian distributed with mean vector $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{\Sigma}$. For a complex scalar a, a^* is the complex conjugate of a. $\mathbf{0}$ and is an all-zero matrix.

2. SYSTEM MODEL

We consider a K-user $M \times N$ MIMO interference channel where K transmitter-receiver pairs exist and transmitters and receivers have N and M antennas, respectively. Due to the interference structure, the received signal at receiver k at time n is expressed as

$$\mathbf{y}_k[n] = \mathbf{H}_{kk}[n]\mathbf{V}_k[n]\mathbf{s}_k[n] + \sum_{l=1,\ l\neq k}^{K} \mathbf{H}_{kl}[n]\mathbf{V}_l[n]\mathbf{s}_l[n] + \mathbf{n}_k[n], \tag{1}$$

where $\mathbf{H}_{kl}[n]$ is the $M \times N$ MIMO channel matrix from transmitter l to receiver k at time n, $\mathbf{V}_l[n]$ and $\mathbf{s}_l[n]$ denote the $N \times d_l$ beamforming matrix and $d_l \times 1$ signal vector, respectively, and $\mathbf{n}_k(n) \sim \mathcal{N}(\mathbf{0}, \sigma_n^2\mathbf{I}_M)$ is the $M \times 1$ additive white Gaussian noise vector.

We assume that the MIMO channels are time-varying. To facilitate the estimation and prediction step, we adopt the state-space model for channel variation and further assume that the channel of a particular transmit-receive antenna pair of a particular transmit-receive user pair is independent of those of others. Thus, the chan-

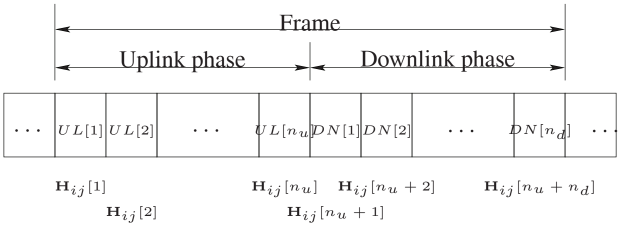

Fig. 1. Frame structure with n_u uplink and n_d downlink symbols

nel is modelled as an m-th order autoregressive (AR) process:

$$H_{kl}^{ij}[n] = \sum_{p=1}^{m} \beta[p] H_{kl}^{ij}[n-p] + u_{kl}^{ij}[n], \tag{2}$$

where $H_{kl}^{ij}[n]$ is the i-th row and j-th column element of $\mathbf{H}_{kl}[n]$ ($1 \le i \le M$, $1 \le j \le N$), $\beta[p]$ and $u_{kl}^{ij}[n]$ $\left(\sim N(0,\sigma_u^2)\right)$ are the AR coefficients and plant noise of the channel process, respectively. It is known that time-varying channels are well modelled by second order AR processes in general [9].

2.1. Pilot Structure and Optimal Estimation

Interference alignment requires the knowledge of the channel at the transmitters. Such knowledge can be obtained in time-division duplex (TDD) systems with pilot symbols, which is assumed in this paper, as shown in Fig. 1. Especially, TDD structure is suitable because of the channel reciprocity, i.e., the downlink and uplink channels are the same at a given time. Thus, pilot symbols are embedded in the uplink stream so that the transmitter can estimate the channel during the uplink phase, and directly uses the estimated channel or optimally predicts the channel during the downlink phase.

The received signal corresponding to uplink pilot symbols is given by

$$\breve{z}_{lk}^{ji}[n] = \breve{H}_{lk}^{ji}[n] p_{lk}^{ji}[n] + \breve{w}_{lk}^{ji}[n], \tag{3}$$

where $\breve{H}_{lk}^{ji}$ is the uplink channel corresponding to the downlink channel H_{kl}^{ij}, $p_{lk}^{ji}[n]$ and $\breve{w}_{lk}^{ji}[n] \sim \mathcal{N}(0,\sigma_w^2)$ are the pilot symbol and noise plus interference, respectively. For the consistency, we rewrite (3) as

$$z_{kl}^{ij}[n] = H_{kl}^{ij}[n] p_{lk}^{ji} + w_{kl}^{ij}[n],\ n = 1, \cdots, n_u, \tag{4}$$

since $\breve{H}_{lk}^{ji}[n] = H_{kl}^{ij}[n]$.

Under the state-space model of (2) and (4) the optimal channel estimation is simply given by Kalman filtering and prediction for the uplink ($n = 1, \cdots, n_u$) and downlink ($n = n_u + 1, \cdots, n_u + n_d$) phases, respectively. For convenience, we here omit the antenna and user indices and rewrite (2) and (4) in vector form. The state-space model is given by

$$\left\{ \begin{array}{lll} \mathbf{x}[n] & = & \mathbf{F}\mathbf{x}[n-1] + \mathbf{u}[n], \\ z[n] & = & \mathbf{C}\mathbf{x}[n] + w[n], \end{array} \right. \tag{5}$$

where $\mathbf{x}[n] = [H[n],\ H[n-1], \cdots, H[n-m+1]]^T$, $\mathbf{u}[n] = [u[n], 0, \cdots,\ 0]^T$, $\mathbf{C} = [1,\ 0, \cdots,\ 0]$, and

$$\mathbf{F} = \left[\begin{array}{cccc} \beta[1] & \beta[2] & \cdots & \beta[m] \\ & \mathbf{I}_{m-1} & & \mathbf{0} \end{array} \right]. \tag{6}$$

Based on this state-space model, standard Kalman filtering and prediction can be applied for channel estimation and prediction.

3. ADAPTIVE BEAM TRACKING

In this section, we present a new adaptive algorithm for beam vector design for interference alignment in time-varying MIMO channels. The proposed algorithm obtains the beam vector solution at time $n+1$ by additive updating the solution at time n instead of calculating the beam vector from the scratch at every time, and thus reduces the complexity of interference alignment significantly in time-varying MIMO channels. The proposed adaptive algorithm is possible due to our recent formulation of interference alignment based on LS approach [7], which is briefly summarized next.

3.1. Background: LS approach to interference alignment [7]

Basically, interference alignment for MIMO interference channels divides the total observation space at the receiver into two subspaces intended for signal and interference (one per each), and projects the received signal onto the orthogonal complement of the interference subspace to make interference-free communication for the desired signal possible. Targeting the maximum achievable DoF of $\frac{KM}{2}$ [2], we here consider the case of $d_1 = \cdots = d_K = d = \frac{M}{2}$. In this case, perfect interference alignment condition is expressed as

$$\mathcal{C}(\mathbf{H}_{12}[n]\mathbf{V}_2[n]) = \mathcal{C}(\mathbf{H}_{13}[n]\mathbf{V}_3[n]) = \cdots = \mathcal{C}(\mathbf{H}_{1K}[n]\mathbf{V}_K[n]), \tag{7}$$

$$\mathcal{C}(\mathbf{H}_{21}[n]\mathbf{V}_1[n]) = \mathcal{C}(\mathbf{H}_{23}[n]\mathbf{V}_3[n]) = \cdots = \mathcal{C}(\mathbf{H}_{2K}[n]\mathbf{V}_K[n]), \tag{8}$$

$$\vdots$$

$$\mathcal{C}(\mathbf{H}_{K1}[n]\mathbf{V}_1[n]) = \mathcal{C}(\mathbf{H}_{K2}[n]\mathbf{V}_2[n]) = \cdots = \mathcal{C}(\mathbf{H}_{KK-1}[n]\mathbf{V}_{K-1}[n]). \tag{9}$$

Instead of using orthogonal complement as in [5], we converted the condition to a system of linear equations with dummy variables using the direct relationship of subspace equivalence [7]. That is, consider the first equality in the first row in (7):

$$\mathcal{C}(\mathbf{H}_{12}[n][\mathbf{v}_1^{(2)}[n], \cdots, \mathbf{v}_d^{(2)}[n]]) = \mathcal{C}(\mathbf{H}_{13}[n][\mathbf{v}_1^{(3)}[n], \cdots, \mathbf{v}_d^{(3)}[n]]), \tag{10}$$

where $\mathbf{v}_m^{(i)}[n]$ is the m-th column of $\mathbf{V}_i[n]$. The equivalence of column spaces spanned by two matrices implies that a column vector in one matrix is represented by a linear combination of the column vectors of the other, and this is a necessary and sufficient condition. Based on this relationship, (10) can be rewritten as

$$\mathbf{H}_{12}[n]\mathbf{v}_1^{(2)}[n] = \alpha_{11}^{(13)}[n]\mathbf{H}_{13}[n]\mathbf{v}_1^{(3)}[n] + \cdots + \alpha_{1d}^{(13)}[n]\mathbf{H}_{13}[n]\mathbf{v}_d^{(3)}[n], \tag{11}$$

$$\vdots$$

$$\mathbf{H}_{12}[n]\mathbf{v}_d^{(2)}[n] = \alpha_{d1}^{(13)}[n]\mathbf{H}_{13}[n]\mathbf{v}_1^{(3)}[n] + \cdots + \alpha_{dd}^{(13)}[n]\mathbf{H}_{13}[n]\mathbf{v}_d^{(3)}[n], \tag{12}$$

where $\alpha_{mm'}^{(13)}[n]$, $m, m' = 1, 2, \cdots, d$, are the coefficients of linear combination. The above multiple equalities can be written in a single matrix-vector form using the Kronecker product as

$$(\mathbf{I}_d \otimes \mathbf{H}_{12}[n]) vec(\mathbf{V}_2[n]) - (\mathbf{A}_{13}[n] \otimes \mathbf{H}_{13}[n]) vec(\mathbf{V}_3[n]) = \mathbf{0}, \tag{13}$$

where $\mathbf{A}_{13}[n] = [\alpha_{mm'}^{(13)}[n]]$. Collecting all equalities generated by (7 - 9), we construct a system of linear equations with dummy variable $\mathbf{A}_{ij}[n]$:

$$\tilde{\mathbf{H}}[n]\mathbf{v}[n] = \mathbf{0} \tag{14}$$

where $\tilde{\mathbf{H}}[n]$ is defined as (15) and has size of $K(K-2)M^2/2 \times KNM/2$, and

$$\mathbf{v}[n] \triangleq [vec(\mathbf{V}_1[n])^T,\ vec(\mathbf{V}_2[n])^T,\ \cdots,\ vec(\mathbf{V}_K[n])^T]^T. \tag{16}$$

When the linear combination coefficients $\{\mathbf{A}_{ij}\}$ and channel information are given, we solve the linear system for interference-aligning beam vectors. The existence of a non-trivial solution

$$\tilde{\mathbf{H}}[n] \triangleq \begin{bmatrix} \mathbf{0} & \mathbf{I}_d \otimes \mathbf{H}_{12}[n] & -\mathbf{A}_{13}[n] \otimes \mathbf{H}_{13}[n] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{I}_d \otimes \mathbf{H}_{12}[n] & \mathbf{0} & -\mathbf{A}_{14}[n] \otimes \mathbf{H}_{14}[n] & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{I}_d \otimes \mathbf{H}_{12}[n] & \mathbf{0} & \cdots & \cdots & -\mathbf{A}_{1K}[n] \otimes \mathbf{H}_{1K}[n] \\ \mathbf{I}_d \otimes \mathbf{H}_{21}[n] & \mathbf{0} & -\mathbf{A}_{23}[n] \otimes \mathbf{H}_{23}[n] & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{I}_d \otimes \mathbf{H}_{21}[n] & \mathbf{0} & \mathbf{0} & -\mathbf{A}_{24}[n] \otimes \mathbf{H}_{24}[n] & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{I}_d \otimes \mathbf{H}_{21}[n] & \mathbf{0} & \mathbf{0} & \cdots & \cdots & -\mathbf{A}_{2K}[n] \otimes \mathbf{H}_{2K}[n] \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{I}_d \otimes \mathbf{H}_{K1}[n] & -\mathbf{A}_{K2}[n] \otimes \mathbf{H}_{K2}[n] & \mathbf{0} & \cdots & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \mathbf{I}_d \otimes \mathbf{H}_{K1}[n] & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{A}_{K(K-1)}[n] \otimes \mathbf{H}_{K(K-1)}[n] & \mathbf{0} \end{bmatrix}, \quad (15)$$

depends on system parameters N, M and K. In overdetermined cases, we apply LS approach given by

$$\hat{\mathbf{v}}[n] = \underset{||\mathbf{v}[n]||=1}{\arg\min} ||\tilde{\mathbf{H}}[n]\mathbf{v}[n]||^2 \quad (17)$$

$||\tilde{\mathbf{H}}[n]\mathbf{v}[n]||^2$ can be regarded as the amount of overall interference miss-alignment. It is known that the solution to (17) is the eigenvector corresponding to the smallest eigenvalue and this solution can be found with the power method [10]. Once $\hat{\mathbf{v}}[n]$ is obtained, we can update $\{\mathbf{A}_{ij}[n]\}$ with $\hat{\mathbf{v}}[n]$ as

$$\begin{aligned} \mathbf{A}_{1j}[n] &= \{(\mathbf{H}_{1j}[n]\hat{\mathbf{V}}_j[n])^\dagger \mathbf{H}_{12}[n]\hat{\mathbf{V}}_2[n]\}^T,\ j = 3, \cdots, K, \quad (18) \\ \mathbf{A}_{ij}[n] &= \{(\mathbf{H}_{ij}[n]\hat{\mathbf{V}}_j[n])^\dagger \mathbf{H}_{i1}[n]\hat{\mathbf{V}}_1[n]\}^T,\ i, j = 2, \cdots, K,\ j \neq i, \end{aligned}$$

since (11 - 12) is rewritten as

$$\mathbf{H}_{12}[n]\mathbf{V}_2[n] = \mathbf{H}_{13}[n]\mathbf{V}_3[n]\mathbf{A}_{13}^T[n],$$

and others can be obtained similarly. Two steps (17) and (18) are iterated for beamforming matrices for interference alinement and the convergence was shown in [7].

3.2. Adaptive interference alignment for time-varying MIMO channels

The LS approach in the previous section provides not only a fast algorithm for beam design for interference alignment in time-invariant MIMO channels but also a basis for the construction of an adaptive algorithm for time-varying channels. In this section, we propose a new adaptive algorithm for interference alignment which updates the beam vector at time n based on the solution at time $n-1$.

First, note that $\tilde{\mathbf{H}}[n]$ in (17) is constructed using $\hat{\mathbf{H}}_{kl}[n]$ and $\mathbf{A}_{kl}[n-1]$. (Refined $\mathbf{A}_{kl}[n-1]$ can be obtained by the iteration discussed in Section 3.1.) Once $\tilde{\mathbf{H}}[n]$ is constructed, (17) can be solved and its solution $\hat{\mathbf{v}}[n]$ is given by the eigenvector associated with the smallest eigenvalue of $\tilde{\mathbf{H}}[n]^H\tilde{\mathbf{H}}[n]$. On the other hand, the channel at time n is represented as

$$\hat{\mathbf{H}}_{kl}[n] = \hat{\mathbf{H}}_{kl}[n-1] + \Delta\mathbf{H}_{kl}[n] \quad (19)$$

assuming reasonable channel fading rate, and thus

$$\begin{aligned} &\tilde{\mathbf{H}}[n](\hat{\mathbf{H}}_{kl}[n], \mathbf{A}_{kl}[n-1]) = \quad (20) \\ &\qquad \tilde{\mathbf{H}}[n-1](\hat{\mathbf{H}}_{kl}[n-1], \mathbf{A}_{kl}[n-2]) + \Delta\tilde{\mathbf{H}}[n]. \end{aligned}$$

Here, the explicit dependency of $\tilde{\mathbf{H}}$ is shown. Using the fact that $\hat{\mathbf{v}}[n]$ is now the eigenvector of $\{\tilde{\mathbf{H}}[n-1] + \Delta\tilde{\mathbf{H}}[n]\}^H\{\tilde{\mathbf{H}}[n-1] + \Delta\tilde{\mathbf{H}}[n]\}$ corresponding to the smallest eigenvalue, we obtain an additive update formula for $\hat{\mathbf{v}}[n]$ based on $\hat{\mathbf{v}}[n-1]$, given in the following theorem.

Theorem 1 *The solution to (17) can be written as*

$$\hat{\mathbf{v}}[n] = \hat{\mathbf{v}}[n-1] + \sum_{i=2}^{KNM/2} \frac{\mathbf{q}_i^H \mathbf{G}_{n-1}[n]\hat{\mathbf{v}}[n-1]}{(\lambda_1 - \lambda_i)}\mathbf{q}_i + o(||\mathbf{G}_{n-1}[n]||^2),$$

where

$$\mathbf{G}_{n-1}[n] \triangleq \frac{\tilde{\mathbf{H}}[n]^H\tilde{\mathbf{H}}[n] - \tilde{\mathbf{H}}[n-1]^H\tilde{\mathbf{H}}[n-1]}{||\tilde{\mathbf{H}}[n]^H\tilde{\mathbf{H}}[n] - \tilde{\mathbf{H}}[n-1]^H\tilde{\mathbf{H}}[n-1]||_2}, \quad (21)$$

and $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_{KNM/2}$ *and* $\mathbf{q}_1 (= \hat{\mathbf{v}}[n-1]), \cdots, \mathbf{q}_{KNM/2}$ *are the ordered eigenvalues and corresponding eigenvectors of* $\tilde{\mathbf{H}}[n-1]^H\tilde{\mathbf{H}}[n-1]$*, respectively.*

Theorem 1 can be proved using the matrix perturbation theory [10]. Due to space limitation, the proof is omitted. Theorem 1 provides a basis for the proposed adaptive algorithm. Note that the full eigen-decomposition of $\tilde{\mathbf{H}}[n-1]^H\tilde{\mathbf{H}}[n-1]$ is not required at each time step. Instead, it can be performed once in a while as in a measurement update after multiple prediction steps in Kalman filtering. In such cases, $\hat{\mathbf{v}}[n+p]$ is obtained from $\hat{\mathbf{v}}[n-1]$, $\{\lambda_i\}$, $\{\mathbf{q}_i\}$, and $\mathbf{G}_{n-1}[n+p]$ for $p = 0, 1, \cdots, L-2$, and the error is in the order of $o(||\mathbf{G}_{n-1}[n+p]||^2)$. Thus, such a scheme is effective when the channel is slowly-varying. The proposed adaptive algorithm is summarized below.

Algorithm 1

- At every $n = iL$ for some integer $L \geq 1$ and $i = 0, 1, \cdots$ $(\mathbf{A}_{kl}^{(0)} = \mathbf{I})$
  - Construct $\tilde{\mathbf{H}}[n]$ with $\{\mathbf{A}_{kl}^{(i)}\}$ and $\{\hat{\mathbf{H}}_{kl}[n]\}$ from the channel estimator in Section 2.1, and update $\{\mathbf{A}_{kl}^{(i)}\}$ by iterating between (17) and (18).
  - Construct $\tilde{\mathbf{H}}[n]$ with the updated $\{\mathbf{A}_{kl}^{(i)}\}$.
  - Obtain $\{\lambda_i\}$ and $\{\mathbf{q}_i\}$ by eigen-decomposition of $\tilde{\mathbf{H}}[n]^H\tilde{\mathbf{H}}[n]$.
  - Set $\hat{\mathbf{v}}[n] = \mathbf{q}_1$. Obtain $\{\hat{\mathbf{V}}_l[n]\}$ by reshaping $\hat{\mathbf{v}}[n]$.
- **for** $n = iL + 1 : (i+1)L - 1$
  - Construct $\tilde{\mathbf{H}}[n]$ using $\{\mathbf{A}_{kl}^{(i)}\}$ and $\{\hat{\mathbf{H}}_{kl}[n]\}$ and construct $\mathbf{G}_{iL}[n]$.
  - Obtain $\hat{\mathbf{v}}[n]$ as

    $$\hat{\mathbf{v}}[n] = \hat{\mathbf{v}}[iL] + \sum_{i=2}^{KNM/2} \frac{\mathbf{q}_i^H \mathbf{G}_{iL}[n]\hat{\mathbf{v}}[iL]}{(\lambda_1 - \lambda_i)}\mathbf{q}_i.$$

  - Obtain $\{\hat{\mathbf{V}}_l[n]\}$ by reshaping $\hat{\mathbf{v}}[n]$.

end

In Algorithm 1, we consider that the update step for $\{\mathbf{A}_{kl}\}$ is also performed every L steps. The frequency of this step can be changed depending on the channel fading rate too. Note that the complexity mostly lies in the operation performed every L time steps. For the intermediate time steps only multiplications and additions are required! Thus, complexity reduction by the proposed adaptive algorithm is significant for large L in slow fading.

4. NUMERICAL RESULTS

In this section, we provide numerical result to verify the adaptive algorithm presented in the previous section. A three user $2(\text{receive}) \times 4(\text{transmit})$ MIMO interference channel was considered. The time-varying channel was generated by the Jakes' model with 2.3 GHz carrier frequency, and the symbol duration was set to 115.2 μs which is one OFDM symbol duration of IEEE 802.16e. The total frame consists of uplink and downlink phases with 25 symbols each. In this setup, the normalized Doppler frequency $f_D T$ is given by 0.0049, $\mathbf{F} = \begin{bmatrix} 1.9994 & -0.9999 \\ 1.0000 & 0.0000 \end{bmatrix}$ and $u[n] \sim \mathcal{N}(0, \sigma_u^2)$ where $\sigma_u^2 = 3.36 \times 10^{-4}$ when mobile speed is 10 km/h. At first, Fig. 2 shows the sum rate without channel prediction. Since the channel is not predicted, beam vectors calculated based on the channel estimate results obtained at the last uplink symbol and these beam vectors are used for all downlink phase. Therefore, the sum rate decreased due to channel error with the mobile speed.

Fig. 3 shows the sum rate performance of the adaptive interference alignment algorithm under the different mobile speed. The value of L was chosen to be equal to n_d. The direct design means that beamforming matrices for interference alignment are directly calculated with the predicted channel matrices at every time step using the iterative LS approach which was shown to yield almost same performance as the iterative method of [5] with low complexity in the considered case of $K = 3, M = 2, N = 4$ [7]. On the other hand, the proposed adaptive algorithm is marked as 'adaptive' in the figure. In both cases, Kalman prediction was used for channel estimation during uplink phase. As shown in Fig. 3, the performance loss by the time-variation of channel increases due to the channel prediction error as the mobile speed increases as expected. Upto 5 km/h, however, the rate loss is negligible. It is seen that the performance gap between the direct method and the proposed adaptive algorithm is negligible. Even if it is not shown in the paper, similar results were obtained for reasonable values of L.

5. CONCLUSIONS

We have considered the problem of interference alignment in time-varying MIMO interference channels. We have proposed an adaptive algorithm for beam vector design based on our previous work of least squares approach to beam design for interference alignment. The proposed algorithm calculates interference-aligning beam vectors by additive update of the previous value and thus reduces complexity significantly. The proposed algorithm is most effective when the number of receive antennas is two due to the limitation of the LS approach on which the proposed algorithm is based [7]. Combined with channel prediction using Kalman filtering, the proposed adaptive algorithm provides an efficient tool to design beam

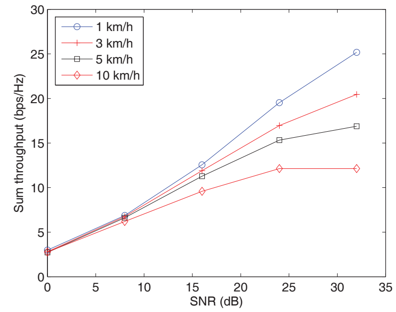

Fig. 2. Sum throughput without channel prediction.

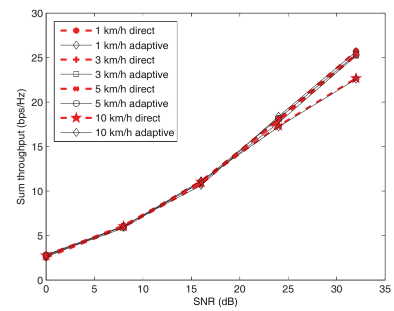

Fig. 3. Sum throughput of direct and adaptive beam design with different mobile speed.

vectors for interference alignment in time-varying MIMO interference channels. As further works, sensitivity to knowledge of $\mathbf{F}$ will be analyzed since this cannot be known in practice.

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