ON OPTIMAL OPERATING CHARACTERISTICS OF SENSING AND TRAINING FOR COGNITIVE RADIOS

Heejung Yu, Youngchul Sung and Yong Hoon Lee

Department of Electrical Engineering
Korea Advanced Institute of Science and Technology, Deajeon, Korea
E-mail:hjyu@stein.kaist.ac.kr; {ysung, yohlee}@ee.kaist.ac.kr

ABSTRACT

The problem of optimal sensing and training in a cognitive radio system is considered when the training signal of the primary transmitter is used for both channel estimation at the primary receiver and sensing for the secondary transmitter. First, the optimal operating characteristics of sensing that maximizes the overall system rate for given training is investigated. It is shown that the optimal false alarm probability at the secondary sensor is monotone increasing as the activity of the primary user increases if the sensing ROC curve is concave. When the primary activity factor is unknown, the max-min criterion is applied to optimal sensing strategy and the resulting max-min optimal solution is given by an equalizer rule for any type of sensing ROC curve. The joint optimization of sensing and training has a unique solution and it can be easily found numerically using a gradient ascent algorithm. By optimal design of sensing and training in such a way, the overall system rate can be improved.

Index Terms – Cognitive radio, optimal sensing, training design, sum rate

1. INTRODUCTION

Due to the scarcity of available frequency bands, cognitive radio has become attractive as a promising technology for the next generation wireless communication [1]. In the cognitive radio, the spectrum utilization is improved by allowing secondary users to access the radio spectrum of primary users in an opportunistic way. In a typical scenario, the access of the secondary user begins with spectrum sensing; the secondary users sense the channel to determine the availability of channel, and access the channel depending on the sensing outcome. The performance of this initial sensing has a big impact on the overall system throughput; in case of false alarm the secondary user's chance of using channel is lost, whereas the primary user's data is in collision when the secondary user miss-detects the channel. Hence, the design of the carrier sensor is one of the important issues in cognitive radio.

The sensing performance depends on various physical-layer parameters and detector type. Several types of sensor can be used. When the signal of the primary user is known, the matched filter can be used. When the primary user's signal is unknown, on the other hand, the energy detector is typically used. However, the performance of the energy detector is inferior to that of the matched filter significantly for the same input signal-to-noise ratio (SNR); it is known that the error rate of the matched filter decays, as a function of SNR, with rate $\exp(-\frac{1}{2}\mathrm{SNR})$ while that of the energy detector decreases with rate $\exp(-\frac{1}{2}\log \mathrm{SNR})$[2, 3]. Moreover, there exists a SNR limit that an energy detector with calibration error can sense [4]. Hence, it is desirable for the secondary user to exploit the known part of the primary user for sensing. In many cases, the known part of the primary user is given by the training signal for various purposes such as channel estimation and power control. It is well known that the training design affects the throughput performance of the primary user [5, 6]. When the secondary users use the training of the primary user for sensing, as we consider, the training design affects not only the throughput of the primary user but also the overall system throughput (including both the primary and secondary users) .

In this paper, we consider such a scenario that the training of the primary user is used for channel estimation at the primary receiver and for sensing at the secondary transmitter. Under the scenario, we investigate the optimal operating characteristics of the sensor at the secondary user and joint optimal design of sensing and training that maximizes the overall system throughput. First, we examine the characteristics of the optimal sensing operation for given training signal. It is shown that the optimal false alarm probability at the secondary sensor is monotone increasing as the activity of the primary user increases if the sensing ROC curve is concave. It is also shown that the optimal sum rate is a convex function of the primary activity factor and the maximum sum rate is achieved when one user occupies the channel all the time under the collision model. When the primary activity factor is unknown, the max-min criterion is applied to optimal sensing strategy and the resulting max-min optimal solution is given by an equalizer rule for any type of sensing ROC curve. The joint optimization problem of sensing and training has a unique solution and it can be readily found numerically using a gradient ascent algorithm.

1.1. Related Works

Training design was extensively surveyed [6]. For example, optimization from the perspective of data rate is presented in [5]. The authors approached the problem using the training-based capacity capturing channel estimation error. The joint design of sensing and rate was investigated by Chen et al. [7]. The authors considered the impact of the carrier sensing under the Markov decision process (MDP) framework, where the transmitter has a control policy of accessing the channel at each slot with probability p. In this paper, we focus on the investigation of the optimal sensing and training design and their properties from the perspective of the overall system rate in a cognitive radio network.

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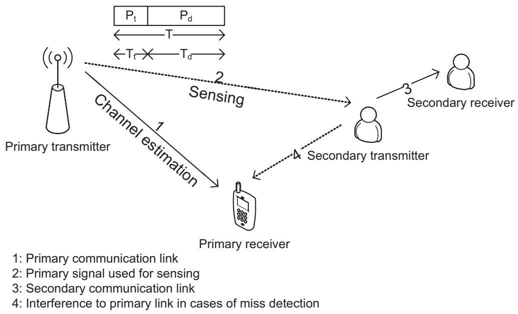

Fig. 1. Cognitive radio system model

2. SYSTEM MODEL

We consider a cognitive radio network with two transmitter-receiver pairs, as shown in Fig. 1, to simplify the problem and gain insights into the inter-relationship between sensing, training and throughput. We assume that transmission is slotted with slot interval of T and the primary and secondary users are synchronized. We assume that for each slot the primary transmitter sends a packet consisting of training signal and data to its receiver independently with probability $\gamma \in [0, 1]$, which is defined as the *primary activity factor*. At the secondary transmitter, the channel is sensed for each slot. If the secondary transmitter does not detect the primary signal, it determines that the channel is available and transmit a packet to its own receiver. Otherwise, it waits until the channel becomes available for its transmission. Here, we assume that the secondary transmitter has always packets to transmit.

2.1. Signal Structure and Training Based Capacity

We assume the time-division multiplexed training signal and data with training signal transmitted first, as shown in Fig. 1. The interval and power of the training part are denoted as T_t and P_t, respectively, while T_d and P_d represent the data interval and data power, respectively. The total time and energy constraints are given by

$$T = T_t + T_d, \quad PT = P_t T_t + P_d T_d, \tag{1}$$

and ρ denotes the portion of total energy allocated to the training, i.e. $P_t T_t = \rho PT$. The received training signal at the primary receiver is given by

$$y[n] = h \left(\sqrt{\frac{\rho PT}{T_t}} t[n] \right) + w[n], \;\; 1 \le n \le T_t, \tag{2}$$

where $t[n]$ and $w[n]$ denote the training signal and additive white Gaussian noise (AWGN) with unit variance. We assume the block fading model in which the channel gain h does not change for one block period.

At the primary receiver, the channel is estimated using the minimum mean square error (MMSE) estimator, and then the training-based capacity of the primary pair is given by [5]

$$C = \frac{T - T_t}{T} \log_2 \left(1 + \frac{P_d \sigma_{\hat{h}}^2}{1 + P_d \sigma_{\tilde{h}}^2} \right) \tag{3}$$

where $\sigma_{\tilde{h}}^2 = \mathbb{E}|h - \hat{h}|^2 = \frac{1}{1+P_t T_t}$ and $\sigma_{\hat{h}}^2 = \mathbb{E}|\hat{h}|^2 = \frac{P_t T_t}{1+P_t T_t}$. In (3), the increase in the training length results in the linear capacity loss, whereas the increase in the training power decreases the capacity in log scale. Hence, the optimal training design is shown to be the one that minimizes the training length (such that the channel is identifiable) and chooses the training energy judiciously [5]. In the single antenna case, therefore, the optimal training scheme is such that the training length is one and the training energy is optimized. For this reason, we focus on the case that $T_t = 1$ and the training power is varied in this paper.

2.2. Sensing: Receiver Operation Characteristics (ROC)

The secondary transmitter senses the channel and transmits its packet only when it determines that the channel is idle. Due to imperfect sensing, however, false alarm and miss detection can occur. We assume that the secondary transmitter senses the primary packet using a matched filter matched to the predetermined training signal of the primary user. Then, the probabilities of the sensing errors depend on the training length and power, and the operating characteristics of the sensor is given by

$$\beta_\rho(\alpha) = Q\left(Q^{-1}(\alpha) - \sqrt{T_t P_t \delta} \right) = Q\left(Q^{-1}(\alpha) - \sqrt{\rho PT\delta} \right) \tag{4}$$

where $Q(\cdot)$ is the Gaussian tail probability, α and $\beta_\rho(\alpha)$ are the false alarm probability and power (i.e., detection probability), respectively, under SNR $P_t\delta$, and δ denotes the relative channel gain difference between link 1 an link 2 in Fig. 1. The operation of the sensor is specified by the receiver operating characteristic (ROC) $(\alpha, \beta_\rho(\alpha))$.

3. OPTIMAL CHARACTERISTICS OF SENSING: OVERALL SYSTEM RATE ANALYSIS

The sensing errors decrease the system throughput; the miss detection causes collision between the primary and secondary links, and the false alarm results in the loss of secondary user's opportunity to use the channel. Here, we assume the collision model between two transmissions from the primary and the secondary transmitters, i.e., no packets are decoded when they collide. Then, the overall system rate is given by

$$C_{sum}(\gamma, \alpha, \rho) = \gamma \beta_\rho(\alpha) C_p + (1-\gamma)(1-\alpha) C_s, \tag{5}$$

where

$$C_p = \frac{T_d}{T} \log_2 \left(1 + \frac{P_d \sigma_{\hat{h}}^2}{1 + P_d \sigma_{\tilde{h}}^2} \right), \tag{6}$$

$$C_s = \log_2 (1 + P_2). \tag{7}$$

Here, we simplify the rate of the secondary pair by using the conventional formula[1], where P_2 denotes the SNR of the secondary pair. Note that the overall system rate is a function of various parameters such as the primary activity factor, training design and the operation of the sensor. The arguments for C_{sum} will be used appropriately if necessary. We first investigate the property of the optimal operating characteristics of sensing for given training design, and then examine the joint optimization of both sensor's operating point and training design.

3.1. Behavior of Optimal Operating Point of Sensor

For a fixed training design, the system rate depends on the operating point $(\alpha, \beta_\rho(\alpha))$ and the primary activity factor γ. For example, Fig. 2 shows the system rate with different sensing operating points and activity factors when $T_t = 1$, $T = 1000$, $\rho = 0.1$, $P = P_2 = 10$ dB and $\delta =$ -30 dB. In the figure, $\gamma = 1$ corresponds to the extreme point in which the primary user always occupies the channel and $(\alpha, \beta_\rho(\alpha)) = (1, 1)$ is the optimal operating

[1] This is not a major issue here.

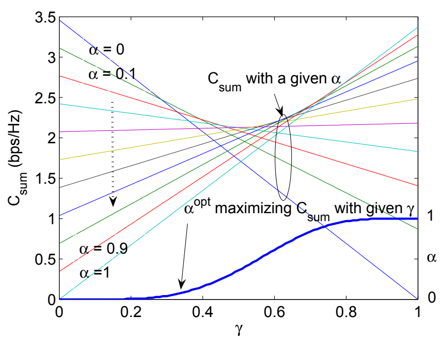

Fig. 2. C_{sum} with different α and γ ($T_t = 1$, $T = 1000$, $\rho = 0.1$ and $P = P_2 = 10$ dB)

point maximizing the primary user's capacity. When the primary activity factor γ is zero, on the other hand, the secondary transmitter can always use the channel and the $(\alpha, \beta_\rho(\alpha)) = (0, 0)$ is the optimal point. The optimal operating point of the sensor in intermediate values of the primary activity factor is characterized in the following proposition.

Proposition 1 *For the intermediate value of $\gamma \in (0, 1)$, there exists an optimal operating $\alpha^{opt}(\gamma)$ when the ROC curve of the sensor is concave, i.e., $\beta_\rho(\alpha)$ is a concave function of α. Furthermore, $\alpha^{opt}(\gamma)$ is non-decreasing in this case as the primary activity factor γ increases. (In the case of strict concavity, it increases monotonically and the optimal value is unique.)*

Proof: The existence is straightforward from the continuity of C_{sum} as a function of α and the finite range of α. Hence, we prove the uniqueness and monotonicity. $\alpha^{opt}(\gamma)$ maximizing C_{sum} is given by solving the following equation:

$$\frac{\partial C_{sum}}{\partial \alpha} = \gamma \frac{\partial \beta_\rho(\alpha)}{\partial \alpha} C_p - (1-\gamma) C_s = 0. \tag{8}$$

$$\frac{\partial \beta_\rho(\alpha)}{\partial \alpha}|_{\alpha=\alpha^{opt}} = \left(\frac{1}{\gamma} - 1\right) \frac{C_s}{C_p}. \tag{9}$$

Due to the concavity of the ROC curve, $\frac{d\beta_\rho(\alpha)}{d\alpha}$ is a non-increasing function. The right hand side of (9) is also a monotonic decreasing function of γ. Therefore, $\alpha^{opt}(\gamma)$ is a non-decreasing function of γ. In case of strict concavity, $\frac{d\beta_\rho(\alpha)}{d\alpha}$ is a monotone decreasing function of α, and the claim follows. ■

The above proposition follows our intuition. When the primary user accesses the channel more actively, the secondary transmitter should allow more false alarm to minimize the miss detection probability. When the channel is not frequently occupied by the primary user, the secondary transmitter should be more aggressive by reducing false alarms. The proposition provides a sufficient condition for such an intuition: *the ROC $(\alpha, \beta_\rho(\alpha))$ is concave*, which is true for many detectors including the matched filter.

Now, we examine the property of the optimal sum rate as a function of γ, defined as

$$C^*(\gamma) \triangleq \max_\alpha C_{sum} = C_{sum}(\gamma, \alpha^{opt}(\gamma)). \tag{10}$$

Proposition 2 *The optimal sum rate $C^*(\gamma)$ (optimized over α for each γ) is a convex function of γ for any type of ROC curve.*

Proof: Let $\gamma_\lambda = \lambda\gamma' + (1-\lambda)\gamma''$ $(0 \le \lambda \le 1)$ and $\bar{\gamma}_\lambda = \lambda\bar{\gamma}' + (1-\lambda)\bar{\gamma}''$, where $\bar{\gamma} = 1 - \gamma$ and similarly for $\bar{\gamma}'$ and $\bar{\gamma}''$. Then, we have

$$\begin{aligned} C^*(\gamma_\lambda) &= \gamma_\lambda \beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \bar{\gamma}_\lambda(1-\alpha^{opt}(\gamma_\lambda))C_s, \\ &= (\lambda\gamma' + (1-\lambda)\gamma'')\beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \\ &\quad (\lambda\bar{\gamma}' + (1-\lambda)\bar{\gamma}'')(1-\alpha^{opt}(\gamma_\lambda))C_s, \\ &= \lambda[\gamma'\beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \bar{\gamma}'(1-\alpha^{opt}(\gamma_\lambda))C_s] + \\ &\quad (1-\lambda)[\gamma''\beta_\rho(\alpha^{opt}(\gamma_\lambda))C_p + \bar{\gamma}''(1-\alpha^{opt}(\gamma_\lambda))C_s], \\ &\le \lambda C^*(\gamma') + (1-\lambda)C^*(\gamma''). \end{aligned}$$

The last step is by the definition of $C^*(\gamma)$. Here, conditions for the ROC curve are not required. ■

The convexity is clearly seen in the upper region in Fig. 2. $C^*(\gamma)$ is given by the curve tangent to all straight lines determined by α in the figure. The convexity of $C^*(\gamma)$ implies that the maximum $C^*(\gamma)$ occurs either at $\gamma = 0$ or $\gamma = 1$. That is, the maximum sum rate of the system is achieved when one user occupies the channel all the time under the collision model [2].

Suppose now that the primary and secondary users have equal priority in the network or the primary activity factor γ is unknown (i.e., the relative transmission activity between the primary and secondary users are unknown to the secondary transmitter). In such cases, the weighted sum rate is not an appropriate criterion any more. To guarantee the equal priority, it is reasonable to maximize the minimum rate of two transmitter-receiver pairs, and the max-min criterion is given by

$$C_{maxmin} = \max_\alpha \{\min(\beta_\rho(\alpha)C_p,\ (1-\alpha)C_s)\}. \tag{11}$$

Proposition 3 *The optimal sensing operating point α^* for the max-min criterion is given by the equalizer rule, i.e.,*

$$\beta_\rho(\alpha^*)C_p = (1-\alpha^*)C_s. \tag{12}$$

Further, this operating point corresponds to that of the primary activity factor γ^* yielding the minimum C_{sum} optimized over α.

Proof: Consider $C_{sum}(\gamma, \alpha)$. For a fixed α, it is a straight line as a function of γ. Hence, for a fixed α the minimum value of C_{sum} over $0 \le \gamma \le 1$ occurs at either $\gamma = 0$ or $\gamma = 1$ with the minimum value of $\min\{\beta_\rho(\alpha)C_p,\ (1-\alpha)C_s\}$. Any straight line that is strictly below the $C^*(\gamma)$ curve does not achieve the max-min criterion since there is a tangent line to $C^*(\gamma)$ (and parallel to that line) that has larger C_{sum}. The straight line tangent to $C^*(\gamma)$ at γ_0 is given by $C_{sum}(\gamma, \alpha^{opt}(\gamma_0))$. Since $C^*(\gamma)$ is convex by Proposition 2 and $\min\{\beta_\rho(\alpha)C_p,\ (1-\alpha)C_s\}$ occurs at $\gamma = 0$ or 1 for straight line $C_{sum}(\gamma, \alpha^{opt}(\gamma_0))$, the max-min is achieved when $C_{sum}(\gamma, \alpha^{opt}(\gamma^*))$ touches $C^*(\gamma)$ and parallel to γ-axis, i.e., $\beta_\rho(\alpha^*)C_p = (1-\alpha^*)C_s$. (See Fig. 2.) γ^* is the primary activity factor yielding the worst weighted sum rate optimized over α. ■

Note that the optimal sensing operating point $\alpha^{opt}(\gamma)$ at the secondary transmitter for the weighed sum rate requires the knowledge of the primary activity factor at the secondary transmitter. The max-min optimal sensing point α^* can be used without the knowledge of γ. In this way, we can maximize the worst data rate between two. Note also in Fig. 2 that max-min point is the minimum point of C_{sum}. At the left side of this point the secondary

[2] This may not be valid if we apply the interference model rather than the collision model

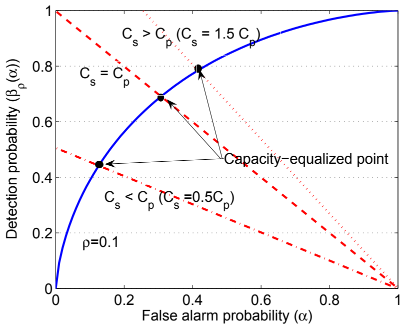

Fig. 3. Max-min solution for different $\frac{C_s}{C_p}$.

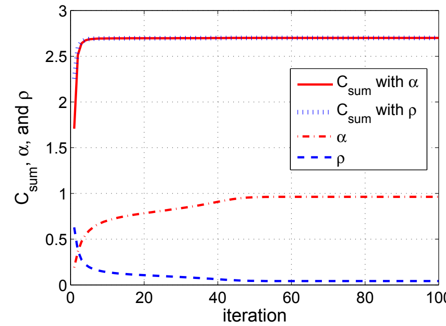

Fig. 5. Learning curves for C_{sum} and α and ρ

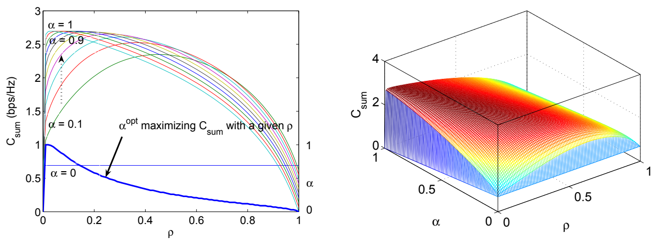

Fig. 4. C_{sum} with different α and ρ when $\gamma = 0.8$.

user has priority while the primary user has priority at the right side of the point. Thus, the max-min sensing operation point corresponds to the sensing operation that equalizes the priorities of the primary and secondary transmitters. The max-min operating point is easily obtained from the sensing ROC, as shown in Fig. 3, by rewriting (12) as $\beta_\rho(\alpha) = \frac{C_s}{C_p}(1-\alpha)$.

3.2. Joint Optimization of Sensing and Training

In the previous section, we optimized the sensing point for given training. Here, we consider joint optimization of both training and sensing to increase the system rate further. Assuming that the primary activity factor is predetermined, we maximize C_{sum} with respect to the sensing operating point α and training power ratio ρ. The optimal solution is given by solving the following optimization problem:

$$\begin{aligned}(\alpha_j^*, \rho_j^*) &= \arg\max_{\alpha,\rho} \gamma\beta_\rho(\alpha)\frac{T-T_t}{T} \\ &\times \log_2\left(1 + \frac{\rho(1-\rho)P^2T^2}{(T-T_t)(1+\rho PT)+(1-\rho)PT}\right) \\ &+ (1-\gamma)(1-\alpha)\log_2(1+P_2)\,. \end{aligned} \tag{13}$$

The optimal solution can be easily found numerically using a gradient ascent algorithm since $C_{sum}(\alpha,\rho)$ is a concave function of α and has a unimode for ρ as a function of ρ for a monotone increasing ROC curve, which can be easily shown using derivatives. Further, due to this property, the solution is unique. Fig. 4 show C_{sum} as a function of α and ρ when $\gamma = 0.8$ and other parameters to be the same as in the example in the previous section.

4. NUMERICAL RESULTS

In this section, we provide a numerical example to illustrate the joint optimization of sensing and training. We consider an iterative gradient ascent algorithm, where ρ is updated for a given α and α is updated for a given ρ and iterate the two steps until it converges. Fig. 5 shows the convergence of α and ρ in this algorithm. We use $\gamma = 0.8$, $P = P_2 = 10$ dB, $T = 1000$, $T_1 = 1$, and $\delta =$ -30 dB. The false alarm probability α converges to the optimal solution $\alpha^{opt} = 0.96$, and the training power ρ converges to the optimal solution $\rho^{opt} = 0.042$. The result coincides with the direct solution of (13) with the considered parameters.

5. CONCLUSIONS

We have considered the problem of optimal sensing and training in a cognitive radio system, where the training signal of the primary transmitter is used for both channel estimation at the primary receiver and sensing for the secondary transmitter. We have investigated the optimal operating characteristics of sensing that maximizes the overall system rate, revealing that the optimal operating point depends on various parameters and the optimal solution is given. The joint optimization problem of sensing and training has a unique solution and it can be found numerically using a gradient ascent algorithm. By optimal design of sensing and training in such a way, the overall system rate can be improved. Future works include the extension to multiple secondary user cases.

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