Adaptive Interference Suppression in MIMO Multiple Access Channels Based on Dual-Domain Approach

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***Abstract*— In this paper, the problem of adaptive filter design for interference suppression in multiple-input multiple-output (MIMO) multiple access (MAC) channels is considered. A new receiver filter is proposed by formulating the problem as quadratic minimization under double constraints: One is the widely-used distortionless constraint and the other is an additional constraint that bounds the energy of interfering signals remaining in the filter output. An efficient adaptive algorithm based on the dual-domain approach is presented to implement the proposed receiver filter. The proposed adaptive algorithm effectively incorporates side information into filter update, and yields better performance than existing adaptive algorithms. The simulation results show the efficacy of the proposed adaptive algorithm.**

I. Introduction

The beamformer design problem has been investigated extensively for several decades due to its wide applications in many fields including array processing, spatial filtering, interference suppression, smart antenna systems, etc. The most well-known classical beamformers include Capon and zero-forcing (ZF) beamformers (see e.g., [1]) both of which can be formulated as the linearly constrained minimum variance (LCMV) design [2]. Under the distortionless constraint on the desired signal, the Capon (i.e., minimum variance distortionless response (MVDR)) beamformer achieves the minimum mean square error (MMSE) with the knowledge of the steering vector of the desired signal. On the other hand, the ZF beamformer utilizes the steering vector information of other interfering signals also to null out the other interfering signals perfectly, but does not achieve MMSE due to noise enhancement. In the previous work [3] an efficient adaptive beamforming algorithm based on the direction of arrival information of all signals has been proposed in a single-input multiple-output (SIMO) setup by applying the multi-domain adaptive filtering technique [4].

In this paper, we extend the algorithm to MIMO systems and also introduce a new beamformer design criterion to provide a new insight into the beamformer design and corresponding adaptive algorithm in [3]. The new design criterion is given by quadratic cost minimization under linear equality and quadratic inequality constraints, which is a standard optimization problem, and the proposed criterion contains both Capon and ZF beamformers as special cases. For general cases, the proposed beamformer resides between the two extreme beamformers. While the classical LCMV beamformers are readily implemented by conventional methods like the constrained (normalized) least mean square (LMS) algorithm, such conventional adaptive algorithms do not render an easy solution for the adaptive implementation of the proposed beamformer containing a quadratic inequality constraint. To circumvent this difficulty, we resort to the dual-domain approach [4] which can incorporate the quadratic constraint effectively by introducing a secondary domain that can be linearly transformed from the primary domain. Although the closed-form solution of the proposed design can not outperform that of the optimal Capon beamformer, the adaptive implementation of the proposed beamformer based on the dual-domain approach provides significant performance gain over conventional adaptive algorithms realizing the Capon beamformer since the dual-domain approach incorporates side information effectively.

The remainder of the paper is organized as follows. In Section II the data model is introduced. Our new design criterion and its adaptive algorithm are provided in Sections III and IV, respectively. Numerical results are provided in Section V, followed by conclusion in Section VI.

II. Data Model

We consider a MIMO multiple access channel in which there exist K transmitters (or users) each of which is equipped with N transmit-antennas and a single receiver (typically a base station) equipped with M receive-antennas, as shown in Fig. 1. We assume that the channels are deterministic and known to the receiver exactly or roughly. Given a signal vector $\boldsymbol{s}_i[n] \in \mathbb{C}^N$ of the nth symbol time at the ith transmitter, the received signal vector is given as follows:

$$\boldsymbol{y}[n] = \boldsymbol{H}_1\boldsymbol{s}_1[n]+\boldsymbol{H}_2\boldsymbol{s}_2[n]+\cdots+\boldsymbol{H}_K\boldsymbol{s}_K[n]+\boldsymbol{n}[n] \in \mathbb{C}^M,$$

where $\boldsymbol{H}_i \in \mathbb{C}^{M\times N}$ is the MIMO channel matrix from the ith transmitter to the receiver; its (p,q) element corresponds to the channel from the qth transmit-antenna to the pth receive-antenna, and $\boldsymbol{n}[n] \in \mathbb{C}^M$ is an independent and identically distributed (i.i.d.) complex Gaussian noise vector. For simplicity, $\boldsymbol{s}_i$, $i = 1,\cdots,K$, are assumed independent to each other, and $\boldsymbol{s}_i \sim \mathcal{N}(0,\sigma_s^2\boldsymbol{I})$, where σ_s^2 is the signal

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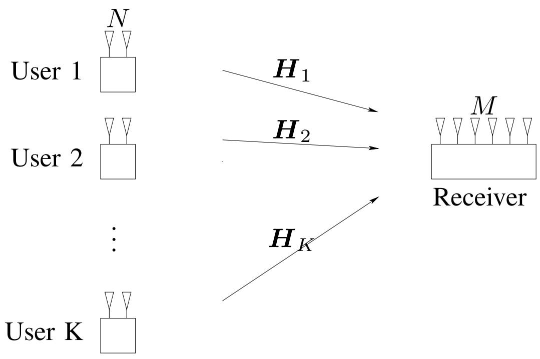


Fig. 1. System model.

power and $\boldsymbol{I}$ the identity matrix. The problem is to recover $\boldsymbol{s}_i$ from the data $\boldsymbol{y}[n]$ acquired at the receiver. A linear receiver approaches this task by designing a matrix $\boldsymbol{F}_i \in \mathbb{C}^{M\times N}$ for each user i such that $\hat{\boldsymbol{s}}_i[n] := \boldsymbol{F}_i^{\mathsf{H}}\boldsymbol{y}[n] \approx \boldsymbol{s}_i[n]$, where $(\cdot)^{\mathsf{H}}$ stands for *Hermitian transpose*. We use the notation $\|\cdot\|$ to denote the norm defined as $\|\boldsymbol{x}\| := \sqrt{\boldsymbol{x}^{\mathsf{H}}\boldsymbol{x}}$ for $\boldsymbol{x}$ in any dimensional complex vector space. For a matrix $\boldsymbol{X}$, we use $\|\boldsymbol{X}\|_{\mathrm{F}}$ to denote the Frobenius norm.

III. The Proposed Receiver Design Criterion

In this section we present our new criterion for beamformer design and provide its relationship to existing criteria such as Capon and ZF beamforming methods. (For simplicity we drop the index n in this section.) Our new design criterion to recover $\boldsymbol{s}_i$ is given by

$$\begin{aligned}\boldsymbol{F}_{i,\mathrm{RZF}} := \ & \underset{\boldsymbol{F}\in\mathbb{C}^{M\times N}}{\arg\min}\ \mathsf{E}\left\{\left\|\boldsymbol{F}^{\mathsf{H}}\boldsymbol{y}\right\|^2\right\} \text{ such that} \\ & \text{(C.1)} \quad \boldsymbol{H}_i^{\mathsf{H}}\boldsymbol{F} = \boldsymbol{I} \quad \text{and} \\ & \text{(C.2)} \quad \left\|\tilde{\boldsymbol{H}}_i^{\mathsf{H}}\boldsymbol{F}\right\|_{\mathrm{F}}^2 \leq \epsilon, \qquad (1)\end{aligned}$$

where $\mathsf{E}\{\cdot\}$ stands for expectation, $\epsilon \geq 0$, and

$$\tilde{\boldsymbol{H}}_i = [\boldsymbol{H}_1, \cdots, \boldsymbol{H}_{i-1}, \boldsymbol{H}_{i+1}, \cdots, \boldsymbol{H}_K] \in \mathbb{C}^{M\times N(K-1)}.$$

Thus, the proposed scheme tries to minimize the variance or energy of the beamformer output signal while satisfying the distortionless constraint (C.1) and containing other user interference within a certain level (which is imposed by (C.2)). To yield a simpler form for the problem, we define

$$\begin{aligned}\boldsymbol{f} &:= [\boldsymbol{f}_1^{\mathsf{T}}, \boldsymbol{f}_2^{\mathsf{T}}, \cdots, \boldsymbol{f}_N^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{C}^{MN}, \\ \bar{\boldsymbol{y}} &:= [\boldsymbol{y}^{\mathsf{T}}, \boldsymbol{y}^{\mathsf{T}}, \cdots, \boldsymbol{y}^{\mathsf{T}}]^{\mathsf{T}} \in \mathbb{C}^{MN},\end{aligned}$$

where $\boldsymbol{F} =: [\boldsymbol{f}_1, \boldsymbol{f}_2, \cdots, \boldsymbol{f}_N]$ ($(\cdot)^{\mathsf{T}}$ stands for *transpose*). Also we define

$$\begin{aligned}\bar{\boldsymbol{H}}_i &:= \begin{bmatrix} \boldsymbol{H}_i & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{O} & \boldsymbol{H}_i & \cdots & \boldsymbol{O} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{O} & \boldsymbol{O} & \cdots & \boldsymbol{H}_i \end{bmatrix} \in \mathbb{C}^{MN\times N^2}, \\ &= [\bar{\boldsymbol{h}}_{i1}, \cdots, \bar{\boldsymbol{h}}_{iN^2}] \quad \text{(column partition)}\end{aligned}$$

$$\begin{aligned}\bar{\tilde{\boldsymbol{H}}}_i &:= \begin{bmatrix} \tilde{\boldsymbol{H}}_i & \boldsymbol{O} & \cdots & \boldsymbol{O} \\ \boldsymbol{O} & \tilde{\boldsymbol{H}}_i & \cdots & \boldsymbol{O} \\ \vdots & \vdots & \ddots & \vdots \\ \boldsymbol{O} & \boldsymbol{O} & \cdots & \tilde{\boldsymbol{H}}_i \end{bmatrix} \in \mathbb{C}^{MN\times N^2(K-1)}, \\ \boldsymbol{c} &:= [\boldsymbol{e}_1^{\mathsf{T}}, \boldsymbol{e}_2^{\mathsf{T}}, \cdots, \boldsymbol{e}_N^{\mathsf{T}}]^{\mathsf{T}} \in \{0,1\}^{N^2},\end{aligned}$$

where $\boldsymbol{e}_i := [0, \cdots, 0, 1, 0, \cdots, 0]^{\mathsf{T}} \in \{0,1\}^N$, $i = 1, 2, \cdots, N$, stands for the unit vector that has one at the ith position. Then, the proposed receiver in (1) can be rewritten in a vector form as follows:

Problem 1:

$$\begin{aligned}\boldsymbol{f}_{i,\mathrm{RZF}} = \ & \underset{\boldsymbol{f}\in\mathbb{C}^{MN}}{\arg\min}\ \mathsf{E}\left\{\left|\boldsymbol{f}^{\mathsf{H}}\bar{\boldsymbol{y}}\right|^2\right\} = \underset{\boldsymbol{f}\in\mathbb{C}^{MN}}{\arg\min}\ \boldsymbol{f}^{\mathsf{H}}\boldsymbol{R}_{\bar{y}}\boldsymbol{f} \quad \text{s.t.} \\ & \text{(C.1')} \quad \bar{\boldsymbol{H}}_i^{\mathsf{H}}\boldsymbol{f} = \boldsymbol{c} \quad \text{and} \\ & \text{(C.2')} \quad \left\|\bar{\tilde{\boldsymbol{H}}}_i^{\mathsf{H}}\boldsymbol{f}\right\|^2 = \boldsymbol{f}^{\mathsf{H}}\boldsymbol{R}_{\tilde{H}}\boldsymbol{f} \leq \epsilon, \qquad (2)\end{aligned}$$

where $\boldsymbol{R}_{\bar{y}} := \mathsf{E}\{\bar{\boldsymbol{y}}\bar{\boldsymbol{y}}^{\mathsf{H}}\}$ and $\boldsymbol{R}_{\tilde{H}} := \bar{\tilde{\boldsymbol{H}}}_i\bar{\tilde{\boldsymbol{H}}}_i^{\mathsf{H}}$. The matrix $\boldsymbol{R}_{\bar{y}}$ is assumed to be nonsingular due to the presence of noise, so the cost function is strictly convex.

The proposed receiver (1) has interesting properties. First, consider two extreme cases of $\epsilon = 0$ and $\epsilon = \infty$ of our proposed receiver. For $\epsilon = 0$ the proposed receiver reduces to the ZF beamformer, whereas it becomes the Capon (or MVDR) beamformer as $\epsilon \to \infty$. For a finite $\epsilon > 0$ our proposed receiver exists between the two extreme receivers and thus the proposed receiver can be regarded as a restricted minimum output energy (RMOE) receiver or a relaxed ZF (RZF) receiver in this case. We adopt the latter name (RZF) since the proposed receiver exploits the side information $\bar{\tilde{\boldsymbol{H}}}$ as well as the ZF receiver. The cost of Problem 1 is monotonically decreasing as the ϵ value increases, meaning that the minimum cost is achieved by $\epsilon = \infty$ (i.e., by the Capon beamformer). This implies, as well known, that the Capon beamformer yields the minimum mean square error (MMSE) and correspondingly the maximum signal-to-interference-and-noise ratio (SINR). One may therefore think that there would be no reason for considering such an in-between receiver. However, *the proposed receiver has significant advantages in its adaptive implementation, which can be accomplished by the dual-domain approach incorporating the side information softly in adaptive update.*[1] (This will be explained shortly in Section IV.)

Note that the constraints for Capon or ZF beamformers are linear as $\{\bar{\boldsymbol{H}}_i^{\mathsf{H}}\boldsymbol{f} = \boldsymbol{c}\}$ and $\{\bar{\boldsymbol{H}}_i^{\mathsf{H}}\boldsymbol{f} = \boldsymbol{c} \text{ and } \bar{\tilde{\boldsymbol{H}}}_i^{\mathsf{H}}\boldsymbol{f} = \boldsymbol{0}\}$, respectively, and the design problems for Capon and ZF beamformers are convex optimization problems under linear equality constraints. In particular, they belong to quadratic cost minimization under linear equality constraints, which is known as the LCMV problem [2]. On the other hand, the interference constraint (C.2') is nonlinear and the problem

[1] The ZF receiver uses the side information $\bar{\tilde{\boldsymbol{H}}}$ as a hard equality constraint $\bar{\tilde{\boldsymbol{H}}}_i^{\mathsf{H}}\boldsymbol{f} = \boldsymbol{0}$ and yields degraded performance by noise enhancement, whereas the Capon receiver does not require the side information $\bar{\tilde{\boldsymbol{H}}}$.

does not fall into the LCMV category. However the problem is still convex since the quadratic function in (C.2') is convex, although it is not strictly convex, due to the positive semi-definiteness of $\boldsymbol{R}_{\tilde{H}}$. Problem 1 is a convex problem of quadratic cost minimization under linear equalities and quadratic inequality constraints; the feasibility set is the intersection of a linear variety and a 'flat' ellipsoid (due to the singularity of $\boldsymbol{R}_{\tilde{H}}$), hence it is convex. It is well known that for convex optimization problems the strong duality holds if the problem is strictly feasible (which is referred to as *Slater's condition*), and the solution is readily given by Karush-Kuhn-Tucker (KKT) conditions [5]. To satisfy the Slater's condition, we assume $\epsilon \neq 0$. The Lagrangian for Problem 1 is given by

$$L(\boldsymbol{f},\lambda,\nu) = \boldsymbol{f}^{\mathsf{H}}\boldsymbol{R}_{\bar{y}}\boldsymbol{f}+\lambda(\boldsymbol{f}^{\mathsf{H}}\boldsymbol{R}_{\tilde{H}}\boldsymbol{f}-\epsilon)+\sum_{j=1}^{N^2}\nu_j(\bar{\boldsymbol{h}}_{ij}^{\mathsf{H}}\boldsymbol{f}-c_j),$$

where $\lambda \in \mathbb{R}$, $\nu_j \in \mathbb{C}$, and c_j is the jth element of $\boldsymbol{c}$. The KKT conditions are given by (C.1'), (C.2'), $\lambda \geq 0$, together with the complementary slackness and gradient conditions given respectively by

$$\lambda(\boldsymbol{f}^{\mathsf{H}}\boldsymbol{R}_{\tilde{H}}\boldsymbol{f}-\epsilon) = 0 \quad \text{and} \tag{3}$$

$$\boldsymbol{R}_{\bar{y}}\boldsymbol{f}+\lambda\boldsymbol{R}_{\tilde{H}}\boldsymbol{f}+\frac{1}{2}\sum_j \nu_j\bar{\boldsymbol{h}}_{ij} = \boldsymbol{0}. \tag{4}$$

The condition (4) reduces to

$$\boldsymbol{f} = -\frac{1}{2}(\boldsymbol{R}_{\bar{y}}+\lambda\boldsymbol{R}_{\tilde{H}})^{-1}\left(\sum_j \nu_j\bar{\boldsymbol{h}}_{ij}\right) \tag{5}$$

$$= -\frac{1}{2}(\boldsymbol{R}_{\bar{y}}+\lambda\boldsymbol{R}_{\tilde{H}})^{-1}\bar{\boldsymbol{H}}_i\boldsymbol{\nu} \tag{6}$$

and the $\boldsymbol{\nu} := [\nu_1,\cdots,\nu_{N^2}]^{\mathsf{T}}$ is obtained by solving a linear system (obtained from (C.1'))

$$\boldsymbol{A\nu} = \boldsymbol{c}, \tag{7}$$

where $\boldsymbol{A} := -\frac{1}{2}\bar{\boldsymbol{H}}_i^{\mathsf{H}}(\boldsymbol{R}_{\bar{y}}+\lambda\boldsymbol{R}_{\tilde{H}})^{-1}\bar{\boldsymbol{H}}_i$. Thus, we have

$$\boldsymbol{f}_{i,\mathrm{RZF}}^{\lambda} = (\boldsymbol{R}_{\bar{y}}+\lambda\boldsymbol{R}_{\tilde{H}})^{-1}\bar{\boldsymbol{H}}_i\left[\bar{\boldsymbol{H}}_i^{\mathsf{H}}(\boldsymbol{R}_{\bar{y}}+\lambda\boldsymbol{R}_{\tilde{H}})^{-1}\bar{\boldsymbol{H}}_i\right]^{-1}\boldsymbol{c}. \tag{8}$$

Note that the solution (8) is a generalized version of that of the LCMV formulation obtained in [2]. Let $\lambda = 0$ for the complementary slackness (3). In this case, we need to impose (C.2'), i.e.,

$$\left\|\bar{\bar{\boldsymbol{H}}}_i^{\mathsf{H}}\boldsymbol{f}_{i,\mathrm{RZF}}^{0}\right\|^2 \leq \epsilon \tag{9}$$

should be satisfied. In other words, if ϵ is sufficiently large so that (9) is satisfied, then $\boldsymbol{f}_{i,\mathrm{RZF}} = \boldsymbol{f}_{i,\mathrm{RZF}}^{0}$; i.e., the solution reduces back to the LCMV (Capon) beamformer in [2]. On the other hand, when ϵ is small so that (9) is not satisfied, λ can no longer be zero and hence we have

$$\boldsymbol{f}^{\mathsf{H}}\boldsymbol{R}_{\tilde{H}}\boldsymbol{f} = \epsilon \tag{10}$$

for the complementary slackness (this case is more meaningful to us since the interference condition is enforced). The optimal λ is obtained by the condition (10) with $\boldsymbol{f}$ substituted by (8). Note that in this case the norm constraint should be fully exploited for the minimum output energy. $\lambda \to \infty$ implies $\epsilon \to 0$ because in the limit of $\lambda \to \infty$ $(\boldsymbol{R}_{\bar{y}}+\lambda\boldsymbol{R}_{\tilde{H}})^{-1}$ in front of the right-handed side of (8) has the eigenvalue 0 with multiplicity of the rank of $\bar{\bar{\boldsymbol{H}}}_i$, with its corresponding eigenvectors given by any orthonormal basis vectors of the column space of $\bar{\bar{\boldsymbol{H}}}_i$. Thus, $\boldsymbol{f}_{i,\mathrm{RZF}}^{\lambda}$ converges to the ZF beamformer as $\lambda \to \infty$. Although the solution of Problem 1 involves the λ parameter, its adaptive implementation by the dual-domain approach does not explicitly use λ (but solely use ϵ) as shown below.

IV. Adaptive Algorithm Construction

The exact solution to classical Capon or ZF beamformer requires the knowledge of data covariance matrix $\boldsymbol{R}_{\bar{y}}$ in addition to the known channel information $\bar{\boldsymbol{H}}_i$ and $\bar{\bar{\boldsymbol{H}}}_i$, and thus such beamformers are implemented in an adaptive manner in practice. Typically, LCMV beamformers including Capon and ZF methods have been realized by the constrained (normalized) least mean square (LMS) algorithm since [2], in which the LMS adaptation is first applied to reduce the cost and projection to the hyperplane given by the linear constraint is then applied. However, due to our new norm constraint (C.2') the proposed receiver cannot be implemented by classical CNLMS algorithm realizing LCMV beamformers. Thus, we require some new method that can implement the proposed receiver. Our method for an adaptive implementation of the proposed receiver is based on the dual-domain approach [4]. In the dual-domain approach, the second constraint (C.2') in (1) can easily be incorporated into adaptive algorithm construction by constructing a secondary domain. This approach incorporates side information $\bar{\bar{\boldsymbol{H}}}_i$ to attain better performance at the initial phase of adaptation (in which a sufficient amount of data is not yet observed to construct a good approximation of the proposed receiver) or to yield better steady-state performance.

To construct an adaptive algorithm to realize the proposed receiver $\boldsymbol{f}_{i,\mathrm{RZF}}$, we define the following closed convex sets:

$$C := \{\boldsymbol{f} \in \mathbb{C}^{MN} : \bar{\boldsymbol{H}}_i^{\mathsf{H}}\boldsymbol{f} = \boldsymbol{c}\}$$
$$V_n := C \cap \{\boldsymbol{f} \in \mathbb{C}^{MN} : \boldsymbol{f}^{\mathsf{H}}\bar{\boldsymbol{y}}[n] = 0\},\ n \in \mathbb{N}$$
$$B_\epsilon := \{\boldsymbol{x} \in \mathbb{C}^{N^2(K-1)} : \|\boldsymbol{x}\|^2 \leq \epsilon\}$$

where $\boldsymbol{\Sigma} := \bar{\bar{\boldsymbol{H}}}_i/\sigma_{\max}(\bar{\bar{\boldsymbol{H}}}_i) \in \mathbb{C}^{MN\times N^2(K-1)}$ with $\sigma_{\max}(\bar{\bar{\boldsymbol{H}}}_i)$ denoting the maximum singular value of $\bar{\bar{\boldsymbol{H}}}_i$. The meaning of each set will be explained after presenting the algorithm.

Let $\lambda_n \in (0,2)$ denote the step size, and $w_1^{(n)}, w_2^{(n)} \geq 0$ satisfy $w_1^{(n)}+w_2^{(n)} = 1$, $\forall n \in \mathbb{N}$. We set the initial receiver filter as $\boldsymbol{f}_0 := [[\boldsymbol{G}]_1^{\mathsf{T}},[\boldsymbol{G}]_2^{\mathsf{T}},\cdots,[\boldsymbol{G}]_N^{\mathsf{T}}]^{\mathsf{T}} \in C$, where $[\boldsymbol{G}]_j$ stands for the jth column of $\boldsymbol{G} := \boldsymbol{H}_i(\boldsymbol{H}_i^{\mathsf{H}}\boldsymbol{H}_i)^{-1}$. The proposed algorithm generates the sequence of filters $(\boldsymbol{f}_n)_{n\in\mathbb{N}}$

recursively as follows.

$$\boldsymbol{f}_{n+1} := P_C\left(\boldsymbol{f}_n + \lambda_n\mu_n(w_1^{(n)}\boldsymbol{g}_n^{(1)} + w_2^{(n)}\boldsymbol{\Sigma}\boldsymbol{g}_n^{(2)})\right),\ n \in \mathbb{N},$$

where

$$\boldsymbol{g}_n^{(1)} := P_{V_n}(\boldsymbol{f}_n) - \boldsymbol{f}_n \in \mathbb{C}^{MN} \quad (11)$$

$$\boldsymbol{g}_n^{(2)} := P_{B_\epsilon}(\boldsymbol{\Sigma}^{\mathsf{H}}\boldsymbol{f}_n) - \boldsymbol{\Sigma}^{\mathsf{H}}\boldsymbol{f}_n \in \mathbb{C}^{N^2(K-1)} \quad (12)$$

$$\mu_n := \begin{cases} \dfrac{w_1^{(n)}\left\|\boldsymbol{g}_n^{(1)}\right\|^2 + w_2^{(n)}\left\|\boldsymbol{g}_n^{(2)}\right\|^2}{\left\|w_1^{(n)}\boldsymbol{g}_n^{(1)} + w_2^{(n)}\boldsymbol{\Sigma}\boldsymbol{g}_n^{(2)}\right\|^2} \\ \text{if } \boldsymbol{f}_n \notin V_n \cap (\boldsymbol{\Sigma}^{\mathsf{H}})^{-1}(B_\epsilon) \\ 1 \text{ otherwise.} \end{cases} \quad (13)$$

Here, for any set S, we denote by P_S the metric projection onto S, and $(\boldsymbol{\Sigma}^{\mathsf{H}})^{-1}(B_\epsilon) := \left\{\boldsymbol{x} \in \mathbb{C}^N : \boldsymbol{\Sigma}^{\mathsf{H}}\boldsymbol{x} \in B_\epsilon\right\}$. For the detailed properties of the proposed algorithm, the reader may refer to [4]. Below we only describe its intuitive idea. By performing P_C as the final operation of updating the filter at each iteration, the (absolute) constraint $\boldsymbol{f} \in C$ is imposed, keeping the desired signal $\boldsymbol{s}_i[n]$ undistorted. The projection P_{V_n} itself enforces the output energy to be zero, but because of the relaxation by $\lambda_n\mu_n$ (which is typically not unity) and its following P_C operation, the output energy is reasonably recovered to maintain the signal power. The same applies to P_{B_ϵ}, which itself enforces the interference power bounded by ϵ, but it is reasonably relaxed due to its following operations. In this regard, we call $\boldsymbol{f} \in V_n$, or $\boldsymbol{\Sigma}^{\mathsf{H}}\boldsymbol{f} \in B_\epsilon$, as *soft constraint*. Each projection has the following closed-form expression:

$$P_C(\boldsymbol{f}) = \boldsymbol{Q}\boldsymbol{f} + \tilde{\boldsymbol{c}},\ \boldsymbol{f} \in \mathbb{C}^{MN}, \quad (14)$$

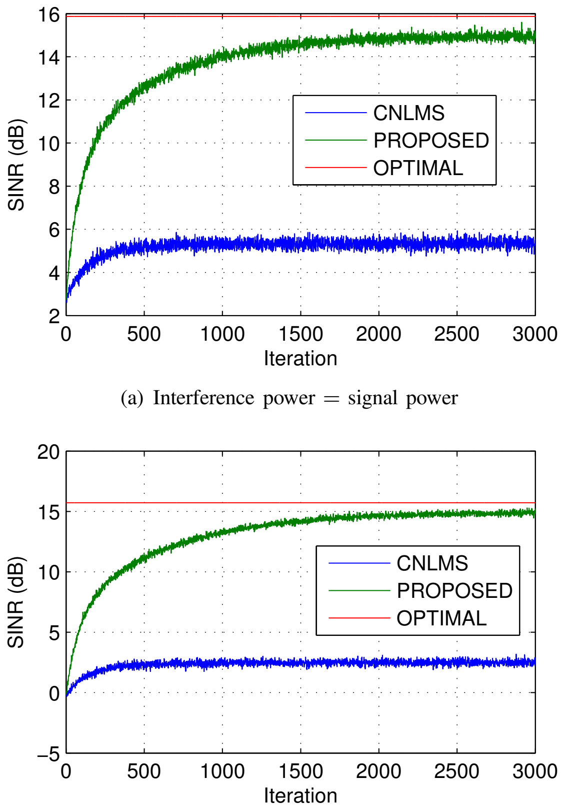
$$P_{V_n}(\boldsymbol{f}) = \boldsymbol{f} - \frac{\langle \boldsymbol{y}[n], \boldsymbol{f}\rangle}{\boldsymbol{y}^{\mathsf{H}}[n]\boldsymbol{Q}\boldsymbol{y}[n]}\boldsymbol{Q}\boldsymbol{y}[n],\ \boldsymbol{f} \in C \subset \mathbb{C}^{MN}, \quad (15)$$

$$P_{B_\epsilon}(\boldsymbol{x}) = \begin{cases} \boldsymbol{x} & \text{if } \|\boldsymbol{x}\|^2 \le \epsilon, \\ \dfrac{\epsilon\boldsymbol{x}}{\|\boldsymbol{x}\|} & \text{otherwise,} \end{cases} \quad \boldsymbol{x} \in \mathbb{C}^{N^2(K-1)}, \quad (16)$$

where $\boldsymbol{Q} := \boldsymbol{I} - \bar{\boldsymbol{H}}_i(\bar{\boldsymbol{H}}_i^{\mathsf{H}}\bar{\boldsymbol{H}}_i)^{-1}\bar{\boldsymbol{H}}_i^{\mathsf{H}}$ and $\tilde{\boldsymbol{c}} := \bar{\boldsymbol{H}}_i(\bar{\boldsymbol{H}}_i^{\mathsf{H}}\bar{\boldsymbol{H}}_i)^{-1}\boldsymbol{c}$.

V. Simulation Results

We set up simulations as follows. There are $K = 4$ transmitters with $N = 2$ antennas and a single receiver with $M = 8$ antennas. The channel matrices are generated randomly from i.i.d. complex Gaussian distribution, and they are then fixed over 500 realizations of the signal and noise vectors ($\boldsymbol{s}_i[n]$ and $\boldsymbol{n}[n]$). For the proposed algorithm, we set $\epsilon := \left\|\boldsymbol{\Sigma}^{\mathsf{H}}\boldsymbol{f}_{i,\mathrm{RZF}}\right\|^2$ (which is the optimum bound), $\lambda_n := 0.03$, and $w_1^{(n)} := w_2^{(n)} := 1/2$. The results are plotted in Fig. 2, in which CNLMS (the blue curve) is the constrained normalized least mean square algorithm that is a normalized version of the Frost's algorithm [1].



(a) Interference power = signal power

(b) Interference power = twice larger than signal power

Fig. 2. SINR curves under SNR 15 dB.

VI. Conclusion

We have proposed a new receiver filter design based on the quadratic optimization problem formulation under the double constraints. We have also presented the dual-domain adaptive algorithm to implement the proposed RZF receiver. The simulation results show that drastic improvements of the performance are achieved due to the use of the side information incorporated into the algorithm in the form of the dual-domain projection. A nontrivial conclusion is that the RZF receiver implemented with the dual-domain adaptive algorithm defeats the Capon beamformer implemented with the CNLMS algorithm. Also we emphasize that the proposed receiver offers a more effective way of exploiting the side information than the ZF receiver.

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